Past-paper walk-through

Algebra ▪ 3.1

eXercise

Questions in this set

- 9709/31 N25 Q10 ▪ 11 marks
- 9709/31 J25 Q1 ▪ 3 marks
- 9709/31 J25 Q5 ▪ 5 marks
- 9709/31 J25 Q10 ▪ 8 marks
- 9709/32 J25 Q2 ▪ 5 marks
- 9709/32 J25 Q10 ▪ 8 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 10

9709/31 N25 ▪ Q10 ▪ 11 marks

Let $f(x) = \dfrac{x^3 + 2x - 11}{(3 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions. **[6]**

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. **[5]**

Solution 10

(a)

Mark scheme

- Dividing (degrees equal): $f(x) = 1 + \dfrac{-3x^2 - 17}{(x+3)(x^2+2)} = 1 - \dfrac{4}{x+3} + \dfrac{x-3}{x^2+2}$.

Solution 10

(b) Mark scheme

- $-\dfrac{4}{x+3} = -\frac{4}{3} + \frac{4}{9}x - \frac{4}{27}x^2$; $\dfrac{x-3}{x^2+2} = -\frac{3}{2} + \frac{1}{2}x + \frac{3}{4}x^2$ (to x^2). So $f(x) = -\frac{11}{6} + \frac{17}{18}x + \frac{65}{108}x^2 + \cdots$

Question 1

9709/31 J25 ▪ Q1 ▪ 3 marks

(a) Sketch the graph of $y = |2x - 3|$. **[1]**

(b) Solve the inequality $3x - 1 < |2x - 3|$. **[2]**

Solution 1

(a)

Mark scheme

- V-shaped graph, symmetric, with vertex at $\left(\frac{3}{2}, 0\right)$, drawn as straight lines and existing for negative x.

Solution 1

(b)

Mark scheme

- Critical value from $3x - 1 = 3 - 2x \Rightarrow x = \frac{4}{5}$; solution $x < \frac{4}{5}$.

Question 5

9709/31 J25 ▪ Q5 ▪ 5 marks

The polynomial $3x^3 + pax^2 + 7a^2x + qa^3$ is denoted by $f(x)$, where p, q and a are constants and $a \neq 0$. When $f(x)$ is divided by $(x + 2a)$ the remainder is $-22a^3$. When $f(x)$ is divided by $(3x - a)$ the remainder is $-a^3$. Find the values of p and q. **[5]**

Solution 5

Mark scheme

- $f(-2a) = -22a^3 \Rightarrow 4p + q = 16$; $f\left(\frac{a}{3}\right) = -a^3 \Rightarrow p + 9q = -31$. Solving: $p = 5,\ q = -4$.

Question 10

9709/31 J25 ▪ Q10 ▪ 8 marks

(a) Find the quotient and remainder when $x^3 + 5x^2 - 2x - 15$ is divided by $x^2 - 3$. **[3]**

(b) The variables x and y satisfy the differential equation $\dfrac{dy}{dx} = \dfrac{x^3 + 5x^2 - 2x - 15}{6y(x^2 - 3)}$. It is given that $y = 2$ when $x = 2$. Solve the differential equation to obtain an expression for y^2 in terms of x. **[5]**

Solution 10

(a)

Mark scheme

- $x^3 + 5x^2 - 2x - 15 = (x^2 - 3)(x + 5) + x$; quotient $x + 5$, remainder x.

Solution 10

(b)

Mark scheme

- $\int 6y\,dy = \int \left(x + 5 + \dfrac{x}{x^2 - 3} \right) dx \Rightarrow 3y^2 = \frac{1}{2}x^2 + 5x + \frac{1}{2}\ln(x^2 - 3) + C$;
  $y = 2, x = 2 \Rightarrow C = 0$. So $y^2 = \frac{1}{6}x^2 + \frac{5}{3}x + \frac{1}{6}\ln(x^2 - 3)$.

Question 2

9709/32 J25 ▪ Q2 ▪ 5 marks

(a) Expand $(6 - x)(1 - 2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. **[4]**

(b) State the set of values of x for which the expansion is valid. **[1]**

Solution 2

(a)

Mark scheme

- $(1-2x)^{-3/2} = 1 + 3x + \frac{15}{2}x^2 + \cdots$, so
  $(6-x)(1-2x)^{-3/2} = 6 + 17x + 42x^2 + \cdots$

Solution 2

(b)

Mark scheme

- Valid for $|x| < \frac{1}{2}$.

Question 10

9709/32 J25 ▪ Q10 ▪ 8 marks

(a) Find the quotient and remainder when x^2 is divided by $1 + 4x^2$. **[2]**

(b) Find the exact value of $\int_0^{0.5} x\tan^{-1}(2x)\,dx$. **[6]**

Solution 10

(a)

Mark scheme

- Quotient $\frac{1}{4}$, remainder $-\frac{1}{4}$ (i.e. $x^2 = \frac{1}{4}(1 + 4x^2) - \frac{1}{4}$).

Solution 10

(b)

Mark scheme

- Integrating by parts and using part (a): $\int_0^{0.5} x \tan^{-1}(2x)\, dx = \frac{\pi}{16} - \frac{1}{8}$.