

Exercise walk-through

Algebra ■ 3.1

eXercise

You must be able to

- split a rational function into partial fractions (including a repeated factor)
- expand $(1 + x)^n$ for fractional/negative n (for $|x| < 1$)
- combine the two (expand after splitting)

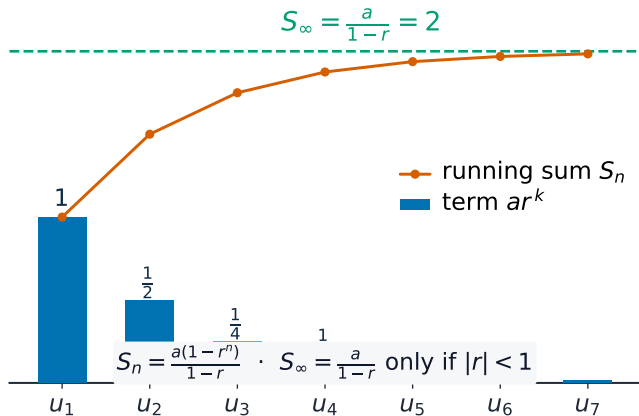
Method — Partial fractions & rational-power binomial

$$\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}: x+4 = A(x-2) + B(x+1); x=2 \Rightarrow B=2;$$

$$x=-1 \Rightarrow A=-1. \text{ So } \frac{2}{x-2} - \frac{1}{x+1}.$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \cdots \text{ for } |x| < 1.$$

Method — Partial fractions & rational-power binomial



A convergent series whose terms shrink to a limit

Practice 2.1 ■ 2 marks

Find the first three terms of $(1 + x)^{-1}$.

Solution 2.1

Worked steps

$$(1+x)^{-1} = 1 - x + x^2 - \dots \quad \text{M1} \quad \text{A1} .$$

Practice 2.2 ■ 3 marks

Write $\frac{5}{(x-1)(x+4)}$ in the form $\frac{A}{x-1} + \frac{B}{x+4}$.

Solution 2.2

Worked steps

$$5 = A(x+4) + B(x-1); x = 1 \Rightarrow A = 1; x = -4 \Rightarrow B = -1 \quad \text{M1} \quad \text{M1}; \frac{1}{x-1} - \frac{1}{x+4} \\ \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The expansion of $(1 + 2x)^{1/2}$ in ascending powers of x begins:

A $1 + x - \frac{1}{2}x^2 + \dots$

B $1 + x + \frac{1}{2}x^2 + \dots$

C $1 + 2x - 2x^2 + \dots$

D $1 - x + x^2 + \dots$

Solution 3.1

A $1 + x - \frac{1}{2}x^2 + \dots$



B $1 + x + \frac{1}{2}x^2 + \dots$

C $1 + 2x - 2x^2 + \dots$

D $1 - x + x^2 + \dots$

Worked steps

A. $(1 + 2x)^{1/2} = 1 + \frac{1}{2}(2x) + \frac{(1/2)(-1/2)}{2}(2x)^2 + \dots = 1 + x - \frac{1}{2}x^2 + \dots$

Exam-style 3.2 ■ 4 marks

Express $\frac{9x + 5}{(x + 1)(2x - 1)}$ in partial fractions.

Solution 3.2

Method: Partial fractions & rational-power binomial

Worked steps

$$9x + 5 = A(2x - 1) + B(x + 1) \quad \text{M1}; \quad x = -1 \Rightarrow -4 = -3A \Rightarrow A = \frac{4}{3}; \quad x = \frac{1}{2} \Rightarrow$$

$$9.5 = 1.5B \Rightarrow B = \frac{19}{3} \quad \text{M1 A1}; \quad \frac{4}{3(x+1)} + \frac{19}{3(2x-1)} \quad \text{A1}.$$

Exam-style 3.3 ■ 3 marks

- (a) Express $\frac{2}{(1-x)(1+2x)}$ in partial fractions.
- (b) Hence find the expansion of $\frac{2}{(1-x)(1+2x)}$ up to the term in x^2 .

Solution 3.3

Method: Partial fractions & rational-power binomial

Worked steps

$$(a) \ 2 = A(1 + 2x) + B(1 - x); \ x = 1 \Rightarrow A = \frac{2}{3}; \ x = -\frac{1}{2} \Rightarrow B = \frac{4}{3} \quad \text{M1 M1};$$

$$\frac{2/3}{1-x} + \frac{4/3}{1+2x} \quad \text{A1}.$$

$$(b) \ \frac{2}{3}(1 + x + x^2) + \frac{4}{3}(1 - 2x + 4x^2) \quad \text{M1 M1}; = \frac{2}{3} + \frac{4}{3} + \left(\frac{2}{3} - \frac{8}{3}\right)x + \left(\frac{2}{3} + \frac{16}{3}\right)x^2$$

$$\text{M1}; = 2 - 2x + 6x^2 \quad \text{A1}.$$