Exercise walk-through

Integration ▪ 2.5

eXercise

You must be able to

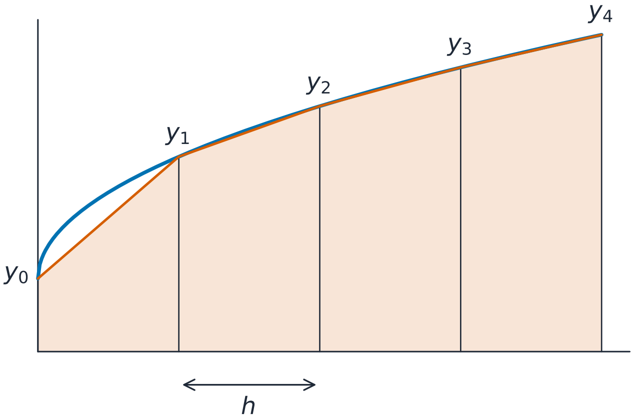
- integrate e^{ax+b}, $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$
- integrate $\sin^2/\cos^2$ using a double-angle identity
- estimate a definite integral with the trapezium rule

Method — Powers of sin & the trapezium rule

$\int 6\sin^2 x\,dx$ — use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$:

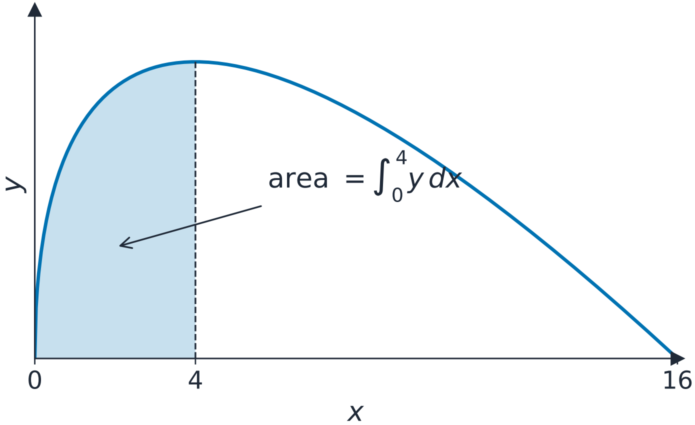
$$\int (3 - 3\cos 2x)\,dx = 3x - \frac{3}{2}\sin 2x + C.$$

Method — Powers of sin & the trapezium rule



The area under a curve split into trapezium strips of width h

Method — Powers of sin & the trapezium rule



The trapezium rule estimates this area under the curve

Practice 2.1 ▪ 2 marks

Find $\displaystyle\int e^{2x+1}\,dx$.

Solution 2.1

Worked steps

$\frac{1}{2}e^{2x+1} + C$ **M1** **A1**.

Practice 2.2 ▪ 2 marks

Find $\displaystyle\int \frac{1}{2x+3}\,dx$.

Solution 2.2

Worked steps

$\frac{1}{2}\ln|2x+3|+C$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$\int \cos 4x \, dx$ equals:

A $4\sin 4x + C$

B $\frac{1}{4}\sin 4x + C$

C $-\frac{1}{4}\sin 4x + C$

D $-4\sin 4x + C$

Solution 3.1

A $4\sin 4x + C$

B $\frac{1}{4}\sin 4x + C$ ✓

C $-\frac{1}{4}\sin 4x + C$

D $-4\sin 4x + C$

Worked steps

B. $\int \cos 4x\, dx = \frac{1}{4}\sin 4x + C$.

Exam-style 3.2 ▪ 4 marks

Find $\int_0^{\pi/4} 4\cos^2 x \, dx$, giving an exact answer.

Solution 3.2

Worked steps

$4\cos^2 x = 2(1 + \cos 2x)$ **M1**; $\displaystyle\int_0^{\pi/4} (2 + 2\cos 2x)\, dx = [2x + \sin 2x]_0^{\pi/4}$ **M1**;
$= \left(\frac{\pi}{2} + 1\right) - 0 = \frac{\pi}{2} + 1$ **A1** **A1**.

Exam-style 3.3 ▪ 4 marks

Use the trapezium rule with **four** strips to estimate $\int_0^2 \sqrt{1+x^3}\,dx$, giving your answer to 3 s.f.

Solution 3.3

Method: Powers of sin & the trapezium rule

Worked steps

$h = 0.5$; y at $x = 0, 0.5, 1, 1.5, 2$: 1, $\sqrt{1.125} = 1.0607$, $\sqrt{2} = 1.4142$, $\sqrt{4.375} = 2.0917$, $\sqrt{9} = 3$ **M1** *ordinates*; $\int \approx \frac{0.5}{2}[1 + 3 + 2(1.0607 + 1.4142 + 2.0917)]$ **M1**; $= 0.25[4 + 2(4.5666)] = 0.25(13.133) = 3.28$ (3 s.f.) **M1** **A1**.