Exercise walk-through

Differentiation ▪ 2.4

eXercise

You must be able to

- differentiate e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$
- apply the product and quotient rules
- differentiate parametric and implicit relations

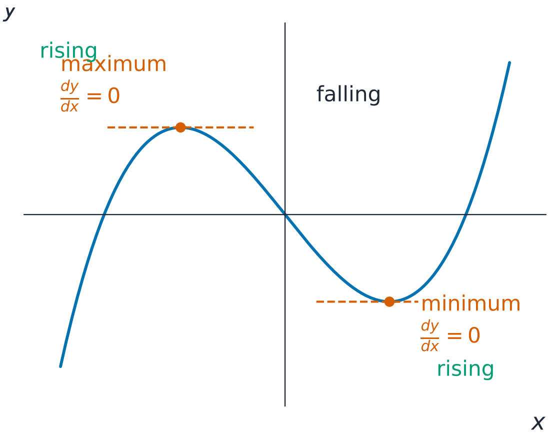
Method — Product rule & implicit

$y = 6x\cos(x^2+1)$. Product rule ($u = 6x$, $v = \cos(x^2+1)$):

$$\frac{dy}{dx} = 6\cos(x^2+1) - 12x^2\sin(x^2+1).$$

Implicit: for $x^2 + y^2 = 25$, differentiate: $2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$.

Method — Product rule & implicit



A curve with a maximum and a minimum, each with a flat tangent

Practice 2.1 ▪ 2 marks

Differentiate $y = x^2 e^x$.

Solution 2.1

Worked steps

$$\frac{dy}{dx} = 2xe^x + x^2e^x = xe^x(2+x)$$ **M1** **A1** *product rule.*

Practice 2.2 ▪ 2 marks

Differentiate $y = \dfrac{\sin x}{x}$.

Solution 2.2

Method: Product rule & implicit

Worked steps

$\frac{dy}{dx} = \frac{x\cos x - \sin x}{x^2}$ **M1** **A1** *quotient rule.*

Exam-style 3.1 ▪ 1 mark

$\dfrac{d}{dx}(\ln(3x))$ equals:

A $\dfrac{1}{3x}$

B $\dfrac{3}{x}$

C $\dfrac{1}{x}$

D $3\ln x$

Solution 3.1

A $\dfrac{1}{3x}$

B $\dfrac{3}{x}$

C $\dfrac{1}{x}$ ✓

D $3\ln x$

Worked steps

C. $\ln(3x) = \ln 3 + \ln x$, so the derivative is $\dfrac{1}{x}$.

Exam-style 3.2 ▪ 4 marks

A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\dfrac{dy}{dx}$ in terms of t, and the value of t at the stationary point(s).

Solution 3.2

Method: Product rule & implicit

Worked steps

$\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2 - 3$ **M1**; $\dfrac{dy}{dx} = \dfrac{3t^2 - 3}{2t}$ **A1**; stationary where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$ **M1** **A1**.

Exam-style 3.3 ▪ 3 marks

A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(b) Find the gradient of the curve at the point $(1, 2)$.

Solution 3.3

Method: Product rule & implicit

Worked steps

(a) differentiate: $2x + y + x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} = 0$ **M1**; $\dfrac{dy}{dx} = -\dfrac{2x+y}{x+2y}$ **M1** **A1**.

(b) at $(1, 2)$: $\dfrac{dy}{dx} = -\dfrac{2+2}{1+4} = -\dfrac{4}{5}$ **M1** **A1**.