# Past-paper walk-through

Trigonometry ▪ 2.3

eXercise

# Questions in this set

- 9709/21 N25 Q3 ▪ 6 marks
- 9709/21 N25 Q4 ▪ 5 marks
- 9709/22 N25 Q2 ▪ 5 marks
- 9709/21 J25 Q7 ▪ 9 marks
- 9709/22 J25 Q7 ▪ 11 marks

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 3

9709/21 N25 ▪ Q3 ▪ 6 marks

**(a)** Solve the equation $|2x-3| = |5x+2|$. **[3]**

**(b)** Hence solve the equation $|2\sec\theta - 3| = |5\sec\theta + 2|$ for $\pi < \theta < 2\pi$. Give your answer correct to 3 significant figures. **[3]**

# Solution 3

## (a)

**Mark scheme**

- Squaring: $(2x-3)^2 = (5x+2)^2 \Rightarrow 21x^2 + 32x - 5 = 0 \Rightarrow x = \frac{1}{7}$ or $x = -\frac{5}{3}$.

# Solution 3

## (b)

### Mark scheme

- $\sec\theta = \frac{1}{7}$ is impossible ($|\cos\theta| \leqslant 1$); $\sec\theta = -\frac{5}{3} \Rightarrow \cos\theta = -\frac{3}{5}$. For $\pi < \theta < 2\pi$, $\theta = 4.07$ rad.

# Question 4

9709/21 N25 ▪ Q4 ▪ 5 marks

Solve the equation $\cot\theta\tan(\theta+45) = 7$ for $0 < \theta < 90$. **[5]**

# Solution 4

## Mark scheme

- With $t = \tan\theta$ and $\tan(\theta + 45) = \frac{t+1}{1-t}$:
  $\frac{1}{t} \cdot \frac{t+1}{1-t} = 7 \Rightarrow 7t^2 - 6t + 1 = 0 \Rightarrow t = \frac{3\pm\sqrt{2}}{7}$. So $\theta = 12.8$ or $32.2$.

# Question 2

9709/22 N25 ▪ Q2 ▪ 5 marks

Solve the equation $2\tan^2\theta + 3\sec\theta = 18$ for $-180 < \theta < 180$. **[5]**

# Solution 2

**Mark scheme**

- $2\sec^2\theta + 3\sec\theta - 20 = 0 \Rightarrow (2\sec\theta - 5)(\sec\theta + 4) = 0 \Rightarrow \cos\theta = \frac{2}{5}$ or $-\frac{1}{4}$. So $\theta = \pm 66.4$ and $\theta = \pm 104.5$.

# Question 7

9709/21 J25 ▪ Q7 ▪ 9 marks

**(a)** Prove that $\sin^2 2x + 4\cos^2 x \cos 2x \equiv 4\cos^4 x$. **[3]**

**(b)** Find the set of possible values of the constant $k$ for which the equation $\sin^2 2x + 4\cos^2 x \cos 2x + 5 = k$ has no real solutions. **[2]**

**(c)** Find the exact value of $\displaystyle\int_{-\frac{1}{3}\pi}^{\frac{1}{3}\pi} \sqrt{\sin^2 t + 4\cos^2\left(\tfrac{1}{2}t\right)\cos t}\, dt$. **[4]**

# Solution 7

## (a)

**Mark scheme**

- $\sin^2 2x + 4\cos^2 x \cos 2x = 4\sin^2 x \cos^2 x + 4\cos^2 x(\cos^2 x - \sin^2 x) = 4\cos^4 x$ (AG).

# Solution 7

**(b)**

**Mark scheme**

- $4\cos^4 x \in [0,4]$, so LHS $+5 \in [5,9]$; no real solutions when $k < 5$ or $k > 9$.

# Solution 7

## (c)

**Mark scheme**

- By part (a) (with $x = \frac{t}{2}$) the integrand $= 2\cos^2\frac{t}{2} = 1 + \cos t$; $\int_{-\pi/3}^{\pi/3}(1+\cos t)\,dt = \left[t + \sin t\right]_{-\pi/3}^{\pi/3} = \frac{2}{3}\pi + \sqrt{3}$.

# Question 7

9709/22 J25 ▪ Q7 ▪ 11 marks

**(a)** Express $4\cos\theta\sin(\theta+30)$ in the form $R\cos(2\theta-\alpha)+k$, where $R>0$, $0<\alpha<90$ and $k$ is a constant. **[6]**

**(b)** Hence solve the equation $12\cos 2\phi\sin(2\phi+30)=5$ for $0<\phi<90$. **[5]**

# Solution 7

## (a)

**Mark scheme**

- $4\cos\theta\sin(\theta+30) = 2\sqrt{3}\cos\theta\sin\theta + 2\cos^2\theta = \sqrt{3}\sin 2\theta + \cos 2\theta + 1 = 2\cos(2\theta - 60) + 1$ (so $R = 2,\ \alpha = 60,\ k = 1$).

# Solution 7

**(b)**

## Mark scheme

- $12\cos 2\phi \sin(2\phi + 30) = 6\cos(4\phi - 60) + 3 = 5 \Rightarrow \cos(4\phi - 60) = \frac{1}{3}$, giving $\phi = 32.6$ and $87.4$.