Exercise walk-through

Trigonometry ▪ 2.3

eXercise

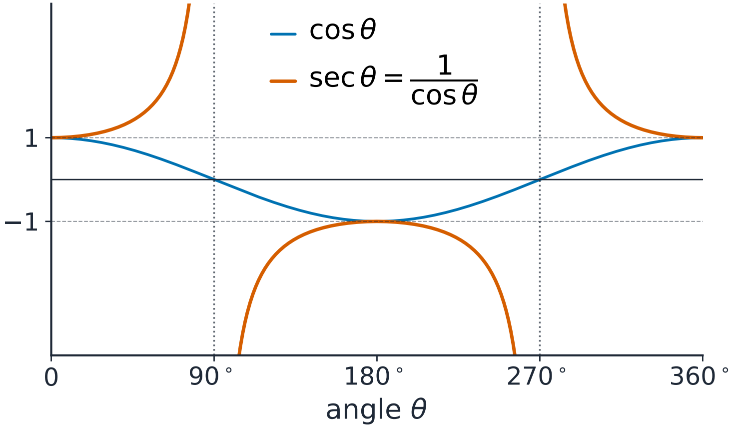
You must be able to

- use $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\csc^2\theta \equiv 1 + \cot^2\theta$
- apply compound- and double-angle formulae
- write $a\sin\theta + b\cos\theta$ as $R\sin(\theta + \alpha)$ and solve

Method — Reciprocal identity & R-formula

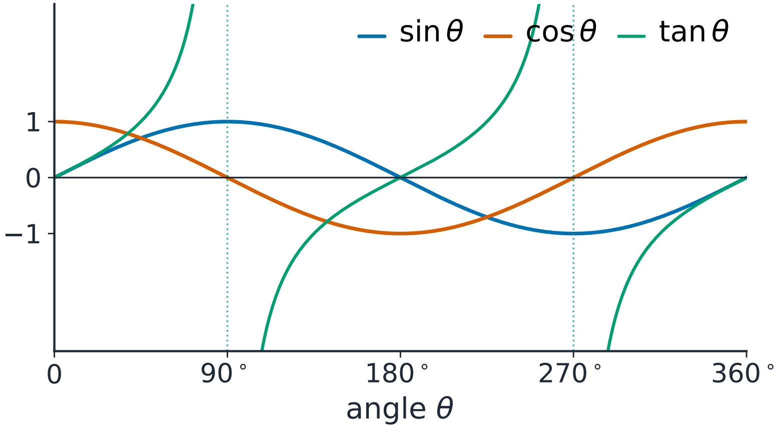
Solve $2\tan^2\theta + 3\sec\theta = 18$ **for** $-180^\circ < \theta < 180^\circ$**.** Use $\tan^2\theta = \sec^2\theta - 1$: $2\sec^2\theta + 3\sec\theta - 20 = 0 \Rightarrow (2\sec\theta - 5)(\sec\theta + 4) = 0$. $\cos\theta = \frac{2}{5}$ or $-\frac{1}{4} \Rightarrow \theta = \pm 66.4^\circ,\ \pm 104.5^\circ$.
R-formula: $a\sin\theta + b\cos\theta = R\sin(\theta + \alpha)$, $R = \sqrt{a^2 + b^2}$, $\tan\alpha = \frac{b}{a}$.

Method — Reciprocal identity & R-formula



The cosine curve and its reciprocal, the secant, rising to asymptotes

Method — Reciprocal identity & R-formula



The underlying sine and cosine graphs

Practice 2.1 ▪ 1 mark

Write $\sec^2\theta$ in terms of $\tan\theta$.

Solution 2.1

Worked steps

$\sec^2\theta = 1 + \tan^2\theta$ **B1**.

Practice 2.2 ▪ 1 mark

Use a double-angle formula to write $\sin 2\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Solution 2.2

Method: Reciprocal identity & R-formula

Worked steps

$\sin 2\theta = 2\sin\theta\cos\theta$ B1.

Exam-style 3.1 ▪ 1 mark

Expressed as $R\sin(\theta+\alpha)$, the value of R for $3\sin\theta + 4\cos\theta$ is:

A 5

B 7

C $\sqrt{7}$

D 12

Solution 3.1

Method: Reciprocal identity & R-formula

A 5 ✓

B 7

C $\sqrt{7}$

D 12

Worked steps

A. $R = \sqrt{3^2 + 4^2} = 5$.

Exam-style 3.2 ▪ 3 marks

Express $5\sin\theta - 12\cos\theta$ in the form $R\sin(\theta - \alpha)$ with $R > 0$ and $0^\circ < \alpha < 90^\circ$.

Solution 3.2

Method: Reciprocal identity & R-formula

Worked steps

$R = \sqrt{5^2 + 12^2} = 13$ M1; $\tan\alpha = \frac{12}{5} \Rightarrow \alpha = 67.4^\circ$ M1; $13\sin(\theta - 67.4^\circ)$ A1.

Exam-style 3.3 ▪ 3 marks

(a) Show that $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} \equiv 2\sec^2\theta$.

(b) Hence solve $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = 8$ for $0^\circ \leqslant \theta \leqslant 180^\circ$.

Solution 3.3

Method: Reciprocal identity & R-formula

Worked steps

(a) common denominator: $\dfrac{(1+\sin\theta)+(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} = \dfrac{2}{1-\sin^2\theta} = \dfrac{2}{\cos^2\theta} = 2\sec^2\theta$ **M1** **M1** **A1**.

(b) $2\sec^2\theta = 8 \Rightarrow \sec^2\theta = 4 \Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{2}$ **M1**; $\theta = 60^\circ$ or 120° **M1** **A1**.