

Exercise walk-through

Logarithmic and exponential functions ■ 2.2

eXercise

You must be able to

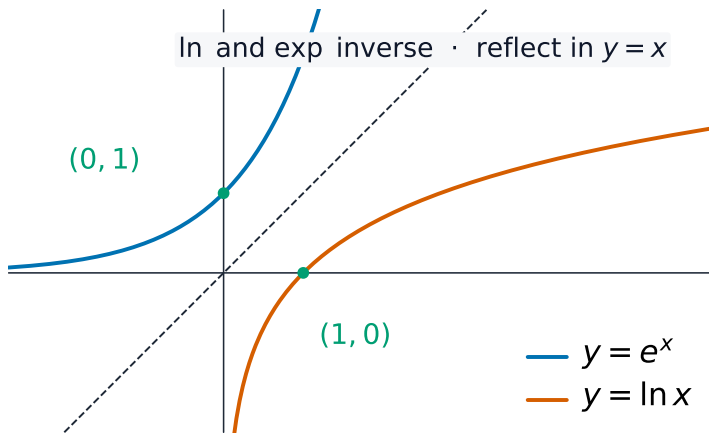
- use the laws of logarithms and $\ln / e^x$ as inverses
- solve equations such as $a^x = b$ by taking logs
- use a log-log (or log-linear) plot to find unknown constants

Method — Solving and linearising

Solve $4^x < 0.05$. $x \ln 4 < \ln 0.05 \Rightarrow x < \frac{\ln 0.05}{\ln 4} = -2.16$ (3 s.f.).

Linear form: $y = Ax^n \Rightarrow \ln y = \ln A + n \ln x$ — plot $\ln y$ against $\ln x$: gradient n , intercept $\ln A$.

Method — Solving and linearising



The curves of e^x and $\ln x$ mirrored across $y = x$

Practice 2.1 ■ 2 marks

Write $\log 12$ in terms of $\log 2$ and $\log 3$.

Solution 2.1

Worked steps

$$12 = 2^2 \times 3, \text{ so } \log 12 = 2 \log 2 + \log 3 \quad \text{M1} \quad \text{A1}.$$

Practice 2.2 ■ 2 marks

Solve $e^{2x} = 7$, giving x to 3 s.f.

Solution 2.2

Worked steps

$$2x = \ln 7 \Rightarrow x = \frac{1}{2} \ln 7 = 0.973 \quad \text{M1} \quad \text{A1}.$$

Exam-style 3.1 ■ 1 mark

The solution of $3^x = 20$ is $x =$

A $\frac{\ln 20}{\ln 3}$

B $\frac{\ln 3}{\ln 20}$

C $\ln \frac{20}{3}$

D $20 - 3$

Solution 3.1

A $\frac{\ln 20}{\ln 3}$



B $\frac{\ln 3}{\ln 20}$

C $\ln \frac{20}{3}$

D $20 - 3$

Worked steps

A. $x \ln 3 = \ln 20 \Rightarrow x = \frac{\ln 20}{\ln 3}.$

Exam-style 3.2 ■ 3 marks

Solve the equation $\ln(2x + 1) + \ln 3 = \ln(4x + 9)$.

Solution 3.2

Worked steps

$$\ln(3(2x + 1)) = \ln(4x + 9) \quad \text{M1}; 6x + 3 = 4x + 9 \Rightarrow 2x = 6 \quad \text{M1}; x = 3 \quad \text{A1}.$$

Exam-style 3.3 ■ 3 marks

The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through $(0, 2)$ is obtained.

- (a) Find the values of n and A .
- (b) Find y when $x = 4$.

Solution 3.3

Method: Solving and linearising

Worked steps

(a) gradient = $n = 1.5$ (B1); intercept $\ln A = 2 \Rightarrow A = e^2 = 7.39$ (M1) (A1).

(b) $y = 7.39 \times 4^{1.5} = 7.39 \times 8 = 59.1$ (M1) (A1).