Past-paper walk-through

Algebra ▪ 2.1

eXercise

Questions in this set

- 9709/21 N25 Q3 ▪ 6 marks
- 9709/21 N25 Q7 ▪ 11 marks
- 9709/22 N25 Q3 ▪ 7 marks
- 9709/22 N25 Q4 ▪ 6 marks
- 9709/21 J25 Q2 ▪ 6 marks
- 9709/21 J25 Q5 ▪ 9 marks
- 9709/22 J25 Q2 ▪ 5 marks
- 9709/22 J25 Q5 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

9709/21 N25 ▪ Q3 ▪ 6 marks

(a) Solve the equation $|2x - 3| = |5x + 2|$. **[3]**

(b) Hence solve the equation $|2\sec\theta - 3| = |5\sec\theta + 2|$ for $\pi < \theta < 2\pi$. Give your answer correct to 3 significant figures. **[3]**

Solution 3

(a)

Mark scheme

- Squaring: $(2x-3)^2=(5x+2)^2 \Rightarrow 21x^2+32x-5=0 \Rightarrow x=\frac{1}{7}$ or $x=-\frac{5}{3}$.

Solution 3

(b)

Mark scheme

- $\sec\theta = \frac{1}{7}$ is impossible ($|\cos\theta| \leqslant 1$); $\sec\theta = -\frac{5}{3} \Rightarrow \cos\theta = -\frac{3}{5}$. For $\pi < \theta < 2\pi$, $\theta = 4.07$ rad.

Question 7

9709/21 N25 ▪ Q7 ▪ 11 marks

The polynomial $p(x)$ is defined by $p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$, where k is a constant. It is given that $(x + 2)$ is a factor of $p(x)$.

(a) Find the value of k. **[2]**

(b) It is given that the equation $p(x) = 0$ has exactly two real roots, denoted by α and β, where α is an integer and β is not an integer. State the value of α and show that β satisfies the equation $x = \sqrt[3]{-2x - 4.5}$. **[4]**

(c) Show by calculation that $-1.4 < \beta < -1.0$. **[2]**

(d) Use an iterative formula, based on the equation in part (b), to find the value of β correct to 3 significant figures. Give the result of each iteration to 5 significant figures. **[3]**

Solution 7

(a)

Mark scheme

- $p(-2) = 32 - 8k + 4k - 34 + 18 = 16 - 4k = 0 \Rightarrow k = 4.$

Solution 7

(b)

Mark scheme

- $\alpha = -2$. Dividing $p(x)$ by $(x+2)$ leaves $2x^3 + 4x + 9 = 0 \Rightarrow x^3 = -2x - 4.5 \Rightarrow x = \sqrt[3]{-2x - 4.5}$, which β satisfies.

Solution 7

(c)

Mark scheme

- $f(x) = 2x^3 + 4x + 9$: $f(-1.4) = -2.09 < 0$ and $f(-1.0) = 3 > 0$, so $-1.4 < \beta < -1.0$.

Solution 7

(d)

Mark scheme

- Iterating $x_{n+1} = \sqrt[3]{-2x_n - 4.5}$ from $x_0 = -1.2$: -1.2806, -1.2475, -1.2610, $-1.2553, \ldots \Rightarrow \beta = -1.26$ (3 s.f.).

Question 3

9709/22 N25 ▪ Q3 ▪ 7 marks

(a) Solve the inequality $|3x-4| \leqslant |2x+5|$. **[4]**

(b) Hence find the largest integer N satisfying the inequality
$|3 \times 7^{0.01N} - 4| \leqslant |2 \times 7^{0.01N} + 5|$. **[3]**

Solution 3

(a)

Mark scheme

- Squaring:
  $(3x-4)^2 \leqslant (2x+5)^2 \Rightarrow 5x^2-44x-9 \leqslant 0 \Rightarrow (5x+1)(x-9) \leqslant 0 \Rightarrow -\frac{1}{5} \leqslant x \leqslant 9.$

Solution 3

(b)

Mark scheme

- Let $t = 7^{0.01N} > 0$; need $t \leqslant 9 \Rightarrow 0.01N \leqslant \dfrac{\ln 9}{\ln 7} = 1.129 \Rightarrow N \leqslant 112.9$. Largest integer $N = 112$.

Question 4

9709/22 N25 ▪ Q4 ▪ 6 marks

The polynomial $p(x)$ is defined by $p(x) = x^4 - 10x^3 + 20x^2 - 30x + 40$.

(a) Find the quotient when $p(x)$ is divided by $(x^2 + 3)$ and show that the remainder is -11. **[3]**

(b) Hence find the real roots of the equation $p(x) + 11 = 0$. Give your answers in exact form. **[3]**

Solution 4

(a)

Mark scheme

- $p(x) = (x^2 + 3)(x^2 - 10x + 17) - 11$. Quotient $x^2 - 10x + 17$, remainder -11.

Solution 4

(b)

Mark scheme

- $p(x) + 11 = (x^2 + 3)(x^2 - 10x + 17) = 0$; $x^2 + 3 = 0$ has no real root, so $x^2 - 10x + 17 = 0 \Rightarrow x = 5 \pm 2\sqrt{2}$.

Question 2

9709/21 J25 ▪ Q2 ▪ 6 marks

(a) Use logarithms to solve the inequality $4^x < 0.05$. Give your answer in the form $x < a$, where the value of a is correct to 3 significant figures. **[2]**

(b) Solve the inequality $|3x + 8| < 9$. **[3]**

(c) Hence state the integers that satisfy both of the inequalities in parts (a) and (b). **[1]**

Solution 2

(a)

Mark scheme

- $x \ln 4 < \ln 0.05 \Rightarrow x < \dfrac{\ln 0.05}{\ln 4} = -2.16$ (3 s.f.).

Solution 2

(b)

Mark scheme

- $3x + 8 = \pm 9 \Rightarrow x = -\frac{17}{3}$ or $\frac{1}{3}$; so $-\frac{17}{3} < x < \frac{1}{3}$.

Solution 2

(c)

Mark scheme

- Integers satisfying both inequalities: $-5,\ -4,\ -3.$

Question 5

9709/21 J25 ▪ Q5 ▪ 9 marks

The polynomial $p(x)$ is defined by $p(x) = ax^3 + bx^2 - ax - 24$, where a and b are constants. It is given that $(2x - 3)$ is a factor of $p(x)$ and that the remainder is -15 when $p(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. **[4]**

(b) Hence factorise $p(x)$ completely. **[3]**

(c) Hence solve the equation $p(3\operatorname{cosec}\theta) = 0$ for $90 < \theta < 270$. **[2]**

Solution 5

(a)

Mark scheme

- $p(\frac{3}{2}) = 0$ and $p(-1) = -15$ give $\frac{27}{8}a + \frac{9}{4}b - \frac{3}{2}a - 24 = 0$ and $-a + b + a - 24 = -15$; solving: $a = 2,\ b = 9$.

Solution 5

(b)

Mark scheme

- $p(x) = 2x^3 + 9x^2 - 2x - 24 = (2x-3)(x^2+6x+8) = (2x-3)(x+2)(x+4)$.

Solution 5

(c)

Mark scheme

- $p(3\operatorname{cosec}\theta) = 0 \Rightarrow 3\operatorname{cosec}\theta = \frac{3}{2},\ -2,\ -4.$ Only $3\operatorname{cosec}\theta = -4 \Rightarrow \sin\theta = -\frac{3}{4}$ is possible, giving $\theta = 228.6.$

Question 2

9709/22 J25 ▪ Q2 ▪ 5 marks

(a) Sketch on the same diagram the graphs of $y = |2x - 9|$ and $y = 4x - 5$. **[2]**

(b) Solve the inequality $|2x - 9| < 4x - 5$. **[3]**

Solution 2

(a)

Mark scheme

- $y = |2x - 9|$ is a V-shape with vertex $(4.5, 0)$ on the positive x-axis; $y = 4x - 5$ is a steeper straight line crossing it.

Solution 2

(b)

Mark scheme

- Relevant branch: $-(2x-9) = 4x-5 \Rightarrow x = \frac{7}{3}$; solution $x > \frac{7}{3}$.

Question 5

9709/22 J25 ▪ Q5 ▪ 9 marks

The polynomial $p(x)$ is defined by $p(x) = ax^4 + bx^3 + 13x^2 - 35x + 15$, where a and b are constants. It is given that $(2x - 1)$ and $(x - 3)$ are factors of $p(x)$.

(a) Find the values of a and b. **[4]**

(b) Hence factorise $p(x)$. **[3]**

(c) Find the least positive value of θ in radians such that $p(\cot 2\theta) = 0$. **[2]**

Solution 5

(a)

Mark scheme

- $p(\frac{1}{2}) = 0$ and $p(3) = 0$ give $\frac{1}{16}a + \frac{1}{8}b = -\frac{3}{4}$ and $81a + 27b = -27$; solving: $a = 2,\ b = -7$.

Solution 5

(b)

Mark scheme

- $p(x) = 2x^4 - 7x^3 + 13x^2 - 35x + 15 = (2x-1)(x-3)(x^2+5)$.

Solution 5

(c)

Mark scheme

- $p(\cot 2\theta) = 0 \Rightarrow \cot 2\theta = \frac{1}{2}$ or 3 (since $x^2 + 5 = 0$ has no real root). Least positive θ: $\cot 2\theta = 3 \Rightarrow \tan 2\theta = \frac{1}{3} \Rightarrow \theta = 0.161$ rad.