Exercise walk-through

Algebra ▪ 2.1

eXercise

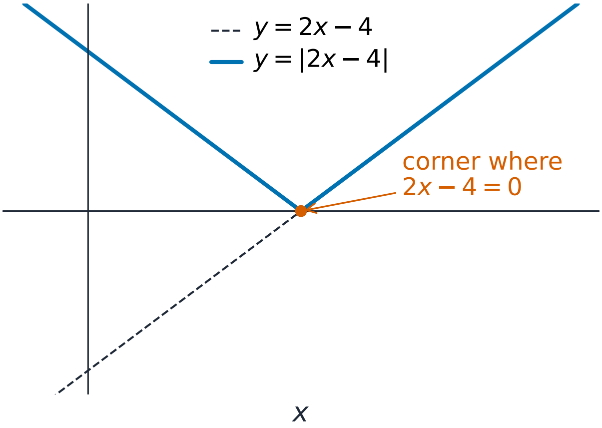
You must be able to

- solve modulus equations and inequalities ($|x-a|<b$, $|a|=|b| \Leftrightarrow a^2=b^2$)
- use the remainder theorem (remainder $= p(a)$)
- use the factor theorem to factorise a polynomial

Method — Modulus & factor theorem

Solve $|3x + 8| < 9$**.** $-9 < 3x + 8 < 9 \Rightarrow -17 < 3x < 1 \Rightarrow -\frac{17}{3} < x < \frac{1}{3}$.
$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$ **has factor** $(x + 2)$**. Find** k**.** $p(-2) = 0$:
$32 - 8k + 4k - 34 + 18 = 0 \Rightarrow 16 - 4k = 0 \Rightarrow k = 4$.

Method — Modulus & factor theorem



A line dipping below the x-axis, folded up into a V

Practice 2.1 ▪ 2 marks

Solve $|2x - 1| = 7$.

Solution 2.1

Worked steps

$2x - 1 = \pm 7$ **M1**; $x = 4$ or $x = -3$ **A1**.

Practice 2.2 ▪ 2 marks

Find the remainder when $x^3 - 2x + 5$ is divided by $(x - 2)$.

Solution 2.2

Worked steps

$p(2) = 8 - 4 + 5 = 9$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$(x-3)$ is a factor of $x^3 - 4x^2 + x + c$. The value of c is:

A -6

B 6

C 12

D -12

Solution 3.1

A -6

B 6 ✓

C 12

D -12

Worked steps

B. $p(3) = 27 - 36 + 3 + c = 0 \Rightarrow c = 6.$

Exam-style 3.2 ▪ 3 marks

Solve the inequality $|x+2| > |x-4|$.

Solution 3.2

Worked steps

square both sides: $(x+2)^2 > (x-4)^2$ **M1**; $x^2+4x+4 > x^2-8x+16 \Rightarrow 12x > 12$ **M1**; $x > 1$ **A1**.

Exam-style 3.3 ▪ 5 marks

The polynomial $p(x) = x^3 + ax^2 + bx + 6$ has a factor $(x - 1)$ and leaves a remainder of 8 when divided by $(x + 1)$.

(a) Form two equations in a and b and solve them.

(b) Hence factorise $p(x)$ completely.

Solution 3.3

Method: Modulus & factor theorem

Worked steps

(a) factor $(x-1)$: $p(1) = 1 + a + b + 6 = 0 \Rightarrow a + b = -7$ **M1**; remainder $p(-1) = -1 + a - b + 6 = 8 \Rightarrow a - b = 3$ **M1**; add the equations: $2a = -4$ **M1**, so $a = -2$, $b = -5$ **A1** **A1**.

(b) $p(x) = x^3 - 2x^2 - 5x + 6$; divide by $(x-1)$ to get $x^2 - x - 6$ **M1**; $x^2 - x - 6 = (x-3)(x+2)$ **M1**; so $p(x) = (x-1)(x-3)(x+2)$ **A1**.