Exercise walk-through

Integration ▪ 1.8

eXercise

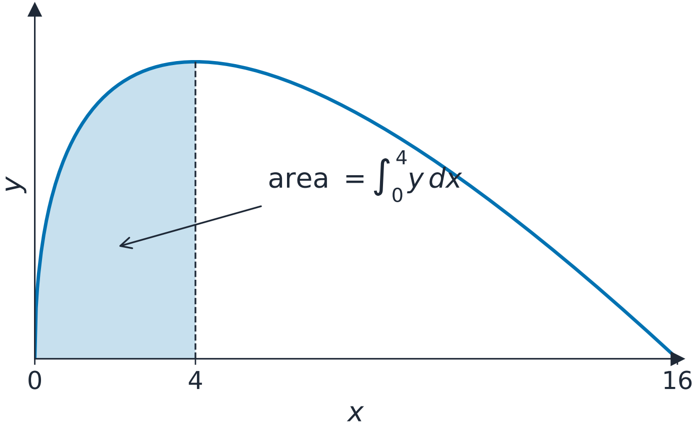
You must be able to

- integrate powers and $(ax + b)^n$, finding $+C$ from a point
- evaluate a definite integral and find an area
- find a volume of revolution with $V = \pi \int y^2 \, dx$

Method — Area and volume

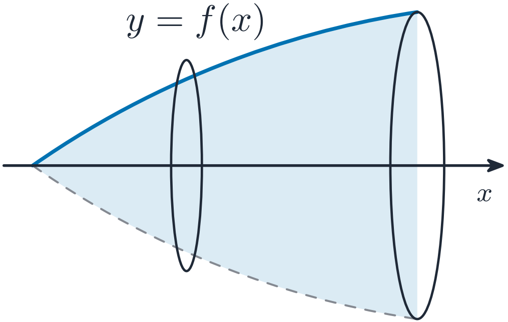
$$\int_0^4 (4x^{1/2} - x)\, dx = \left[\tfrac{8}{3}x^{3/2} - \tfrac{x^2}{2}\right]_0^4 = \tfrac{8}{3}(8) - 8 = \tfrac{40}{3}.$$

Method — Area and volume



A curve with the region between it and the x-axis shaded

Method — Area and volume



rotate the region around the x-axis to sweep a solid

A region rotated about the x-axis to form a solid of discs

Practice 2.1 ▪ 2 marks

Find $\displaystyle\int (6x^2 - 4x + 1)\, dx$.

Solution 2.1

Worked steps

$2x^3 - 2x^2 + x + C$ **M1** **A1**.

Practice 2.2 ▪ 2 marks

Evaluate $\int_1^2 \frac{1}{x^2}\,dx$.

Solution 2.2

Worked steps

$$\left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} - (-1) = \frac{1}{2}$$ **M1** **A1**.

Exam-style 3.1 ▪ 1 mark

$\int_0^3 2x\,dx$ equals:

A 6

B 9

C 12

D 18

Solution 3.1

A 6

B 9 ✓

C 12

D 18

Worked steps

B. $[x^2]_0^3 = 9$.

Exam-style 3.2 ▪ 4 marks

A curve has $\dfrac{dy}{dx} = 3x^2 - 4x$ and passes through $(2, 5)$. Find the equation of the curve.

Solution 3.2

Worked steps

$y = \int (3x^2 - 4x)\,dx = x^3 - 2x^2 + C$ **M1** **A1**; use $(2,5)$: $5 = 8 - 8 + C \Rightarrow C = 5$ **M1**; $y = x^3 - 2x^2 + 5$ **A1**.

Exam-style 3.3 ▪ 3 marks

The region R is bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$.

(a) Find the area of R.

(b) Find the volume when R is rotated through 360° about the x-axis.

Solution 3.3

Method: Area and volume

Worked steps

(a) $\int_0^4 x^{1/2}\,dx = \left[\tfrac{2}{3}x^{3/2}\right]_0^4 = \tfrac{2}{3}(8) = \dfrac{16}{3}$ **M1** **M1** **A1**.

(b) $V = \pi\int_0^4 (\sqrt{x})^2\,dx = \pi\int_0^4 x\,dx = \pi\left[\tfrac{x^2}{2}\right]_0^4 = 8\pi$ **M1** **M1** **A1**.