Exercise walk-through

Differentiation ▪ 1.7

eXercise

You must be able to

- differentiate powers and use the chain rule
- find the equations of a tangent and a normal
- find and classify stationary points (max/min) with $f''(x)$

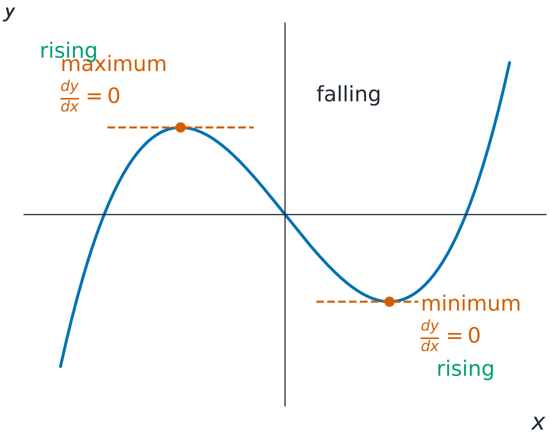
Method — Stationary points

$y = 4x^{1/2} - x$ **has a maximum at** $x = a$**. Find** a**.**

$$\frac{dy}{dx} = 2x^{-1/2} - 1 = 0 \Rightarrow \sqrt{x} = 2 \Rightarrow x = 4, \text{ so } a = 4.$$

Chain rule: $\dfrac{d}{dx}(2x-1)^4 = 4(2x-1)^3 \cdot 2 = 8(2x-1)^3.$

Method — Stationary points



A curve with a maximum and a minimum, each with a flat tangent

Practice 2.1 ▪ 2 marks

Differentiate $y = 3x^4 - \dfrac{2}{x}$.

Solution 2.1

Worked steps

$\dfrac{dy}{dx} = 12x^3 + \dfrac{2}{x^2}$ **M1** **A1** *for each term.*

Practice 2.2 ▪ 2 marks

Find $\dfrac{dy}{dx}$ when $y = (3x + 1)^5$.

Solution 2.2

Worked steps

$\dfrac{dy}{dx} = 5(3x+1)^4 \cdot 3 = 15(3x+1)^4$ **M1** **A1** *chain*.

Exam-style 3.1 ▪ 1 mark

The gradient of $y = x^3 - 2x$ at $x = 2$ is:

A 4

B 6

C 10

D 12

Solution 3.1

A 4

B 6

C 10 ✓

D 12

Worked steps

C. $\dfrac{dy}{dx} = 3x^2 - 2$; at $x = 2$, $= 12 - 2 = 10$.

Exam-style 3.2 ▪ 4 marks

A curve has equation $y = x^3 - 6x^2 + 9x + 1$.

(a) Find the coordinates of the two stationary points.

(b) Determine the nature of each stationary point.

Solution 3.2

Method: Stationary points

Worked steps

(a) $\frac{dy}{dx} = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0$ **M1**; $x = 1, 3$ **A1**; $y(1) = 5$, $y(3) = 1$, so $(1, 5)$ and $(3, 1)$ **A1** **A1**.

(b) $\frac{d^2y}{dx^2} = 6x - 12$; at $x = 1$, $= -6 < 0 \rightarrow$ maximum; at $x = 3$, $= 6 > 0 \rightarrow$ minimum **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

The curve $y = \dfrac{8}{x}$ passes through $P(2, 4)$. Find the equation of the **normal** to the curve at P.

Solution 3.3

Method: Stationary points

Worked steps

$\dfrac{dy}{dx} = -\dfrac{8}{x^2}$; at $x = 2$, gradient $= -2$ **M1**; normal gradient $= \frac{1}{2}$ **M1**; $y - 4 = \frac{1}{2}(x-2) \Rightarrow y = \frac{1}{2}x + 3$ **M1** **A1**.