Exercise walk-through

Series ▪ 1.6

eXercise

You must be able to

- expand $(a+b)^n$ and find a specific term/coefficient
- use the AP and GP formulas for the nth term and the sum
- use $S_\infty = \dfrac{a}{1-r}$ when $|r| < 1$

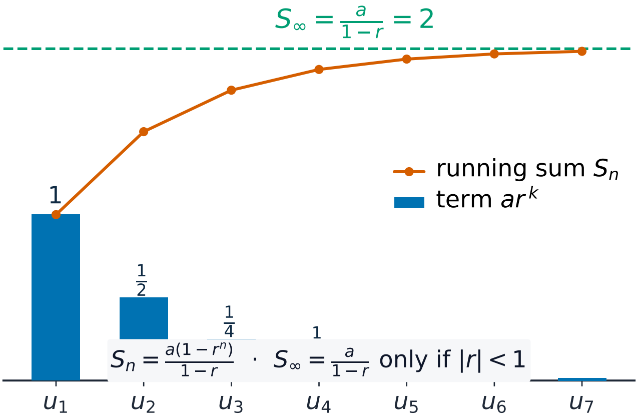
Method — Binomial & GP

First three terms of $(2 - px)^5$:

$$2^5 + \binom{5}{1} 2^4(-px) + \binom{5}{2} 2^3(-px)^2 = 32 - 80px + 80p^2x^2.$$

A GP has third term 18 and $S_3 = 26$, with $r < 0$. From $ar^2 = 18$ and $a(1 + r + r^2) = 26$: $8r^2 - 18r - 18 = 0$, so $r = -\frac{3}{4}$, $a = 32$, and $S_\infty = \dfrac{32}{1 + \frac{3}{4}} = \dfrac{128}{7}$.

Method — Binomial & GP



Shrinking bars whose running sum climbs to the limit a/(1-r)

Practice 2.1 ▪ 2 marks

Find the 10th term of the AP $3, 7, 11, \ldots$.

Solution 2.1

Worked steps

$u_{10} = a + 9d = 3 + 9(4) = 39$ M1 A1.

Practice 2.2 ▪ 1 mark

State the condition on r for a GP to have a sum to infinity.

Solution 2.2

Worked steps

$|r| < 1$ **B1**.

Exam-style 3.1 ▪ 1 mark

The coefficient of x^2 in $(1+2x)^6$ is:

A 12

B 30

C 60

D 15

Solution 3.1

A 12

B 30

C 60 ✓

D 15

Worked steps

C. $\binom{6}{2}(2)^2 = 15 \times 4 = 60.$

Exam-style 3.2 ▪ 3 marks

The first term of a GP is 24 and the sum to infinity is 40.

(a) Find the common ratio.

(b) Find the sum of the first three terms.

Solution 3.2

Method: Binomial & GP

Worked steps

(a) $S_\infty = \dfrac{a}{1-r} = 40 \Rightarrow \dfrac{24}{1-r} = 40 \Rightarrow 1 - r = 0.6$ **M1** **M1**; $r = 0.4$ **A1**.

(b) $S_3 = \dfrac{24(1-0.4^3)}{1-0.4} = \dfrac{24(0.936)}{0.6} = 37.44$ **M1** **A1**.

Exam-style 3.3 ▪ 4 marks

An AP has first term 5 and common difference 3. The sum of the first n terms is 440. Find n.

Solution 3.3

Method: Binomial & GP

Worked steps

$S_n = \frac{n}{2}(2a + (n-1)d) = 440 \Rightarrow \frac{n}{2}(10 + 3(n-1)) = 440$ **M1**; $3n^2 + 7n - 880 = 0$ **M1**; $(3n + 55)(n - 16) = 0$ **M1**; $n = 16$ (reject negative) **A1**.