Past-paper walk-through

Trigonometry ▪ 1.5

eXercise

Questions in this set

- 9709/11 N25 Q5 ▪ 7 marks
- 9709/12 N25 Q6 ▪ 9 marks
- 9709/11 J25 Q1 ▪ 4 marks
- 9709/12 J25 Q5 ▪ 5 marks
- 9709/12 J25 Q7 ▪ 7 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 5

9709/11 N25 ▪ Q5 ▪ 7 marks

(a) Show that $\tan^4\theta - 1 \equiv \dfrac{1 - 2\cos^2\theta}{\cos^4\theta}$. **[3]**

(b) Hence solve the equation $\cos^2\theta(\tan^4\theta - 1) = 7$ for $0 < \theta < 180$. **[4]**

Solution 5

(a)

Mark scheme

- $\tan^4\theta - 1 = \dfrac{\sin^4\theta}{\cos^4\theta} - 1 = \dfrac{(1-\cos^2\theta)^2 - \cos^4\theta}{\cos^4\theta} = \dfrac{1-2\cos^2\theta}{\cos^4\theta}$ (AG).

Solution 5

(b)

Mark scheme

- $\dfrac{1 - 2\cos^2\theta}{\cos^2\theta} = 7 \Rightarrow 9\cos^2\theta = 1 \Rightarrow \cos\theta = \pm\frac{1}{3} \Rightarrow \theta = 70.5$ and 109.5.

Question 6

9709/12 N25 ▪ Q6 ▪ 9 marks

(a) Sketch the graph of $y = 3\sin x + 2$ for $0 \leqslant x \leqslant 2\pi$. **[2]**

(b)(i) $3\sin x + 2 = x$ **[1]**

(b)(ii) $3\sin x + 2 = 5 - x$ **[1]**

(c) Solve the equation $3\sin x + 2 = 5\cos^2 x - 1$ for $0 \leqslant x \leqslant 2\pi$. **[5]**

Solution 6

(a)

Mark scheme

- Curve starts at $(0, 2)$, rises to a maximum $\left(\frac{1}{2}\pi, 5\right)$, falls to a minimum $\left(\frac{3}{2}\pi, -1\right)$, then returns to $(2\pi, 2)$.

Solution 6

(b)(i)

Mark scheme

- 1 solution.

(b)(ii)

Mark scheme

- 3 solutions.

Solution 6

(c)

Mark scheme

- $3\sin x + 2 = 5(1 - \sin^2 x) - 1 \Rightarrow 5\sin^2 x + 3\sin x - 2 = 0 \Rightarrow \sin x = 0.4$ or -1. So $x = 0.41,\ 2.73,\ \frac{3}{2}\pi$.

Question 1

9709/11 J25 ▪ Q1 ▪ 4 marks

Solve the equation $6\sin\theta = 1 + \dfrac{2}{\sin\theta}$ for $-180 < \theta < 180$. **[4]**

Solution 1

Mark scheme

- $6\sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (3\sin\theta - 2)(2\sin\theta + 1) = 0 \Rightarrow \sin\theta = \frac{2}{3}$ or $-\frac{1}{2}$.
  So $\theta = 41.8,\ 138.2,\ -30,\ -150$.

Question 5

9709/12 J25 ▪ Q5 ▪ 5 marks

The equation of a curve is $y = 4\cos 2x + 3$ for $0 \leqslant x \leqslant 2\pi$.

(a) State the greatest and least possible values of y. **[2]**

(b) Sketch the curve. **[2]**

(c) Hence determine the number of solutions of the equation $4\cos 2x + 3 = 2x - 1$ for $0 \leqslant x \leqslant 2\pi$. **[1]**

Solution 5

(a)

Mark scheme

- Greatest value 7; least value -1.

Solution 5

(b)

Mark scheme

- Curve $y = 4\cos 2x + 3$: two complete cycles over $0 \leqslant x \leqslant 2\pi$, starting at the greatest value 7 and initially decreasing, oscillating between $y = -1$ and $y = 7$ (more above the x-axis than below).

Solution 5

(c)

Mark scheme

- 3 solutions.

Question 7

9709/12 J25 ▪ Q7 ▪ 7 marks

(a) Prove the identity $\dfrac{\tan\theta + 7}{\tan^2\theta - 3} \equiv \dfrac{\sin\theta\cos\theta + 7\cos^2\theta}{1 - 4\cos^2\theta}$. **[3]**

(b) Hence solve the equation $\dfrac{\sin\theta\cos\theta + 7\cos^2\theta}{1 - 4\cos^2\theta} = \dfrac{5}{\tan\theta}$ for $0 \leqslant \theta \leqslant 180$. **[4]**

Solution 7

(a)

Mark scheme

- Multiply numerator and denominator of $\dfrac{\tan\theta + 7}{\tan^2\theta - 3}$ by $\cos^2\theta$ (using $\tan\theta = \frac{\sin\theta}{\cos\theta}$): numerator $\to \sin\theta\cos\theta + 7\cos^2\theta$; denominator $\to \sin^2\theta - 3\cos^2\theta = (1 - \cos^2\theta) - 3\cos^2\theta = 1 - 4\cos^2\theta$. AG.

Solution 7

(b)

Mark scheme

- Using part (a): $\tan\theta(\tan\theta + 7) = 5(\tan^2\theta - 3) \Rightarrow 4\tan^2\theta - 7\tan\theta - 15 = 0 \Rightarrow (4\tan\theta + 5)(\tan\theta - 3) = 0$, so $\tan\theta = -1.25$ or 3. Hence $\theta = 71.6$ and $\theta = 128.7$.