

Exercise walk-through

Trigonometry ■ 1.5

eXercise

You must be able to

- recall exact values of $\sin$, $\cos$, $\tan$ for 30° , 45° , 60°
- use $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$
- solve equations (including quadratics in a trig function) over an interval

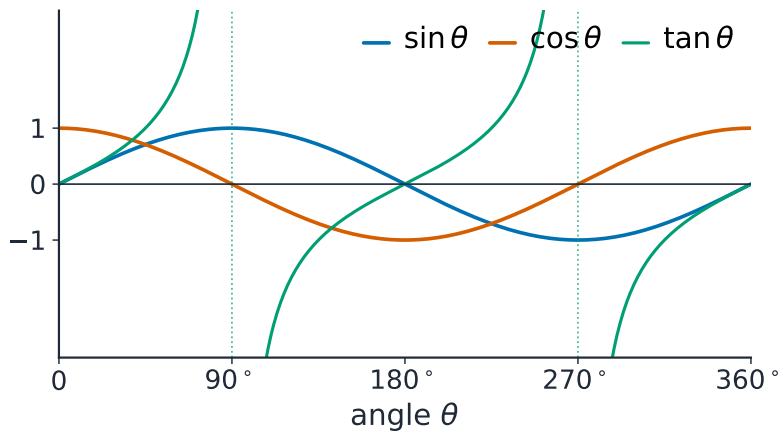
Method — Solving a trig equation

Solve $6 \sin \theta = 1 + \frac{2}{\sin \theta}$ **for** $-180^\circ < \theta < 180^\circ$.

$$\times \sin \theta: 6 \sin^2 \theta - \sin \theta - 2 = 0 \Rightarrow (3 \sin \theta - 2)(2 \sin \theta + 1) = 0.$$

$$\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ; \sin \theta = -\frac{1}{2} \Rightarrow \theta = -30^\circ, -150^\circ.$$

Method — Solving a trig equation



Graphs of sine, cosine and tangent over 0 to 360 degrees

Practice 2.1 ■ 1 mark

Write down the exact value of $\cos 30^\circ$.

Solution 2.1

Worked steps

$$\frac{\sqrt{3}}{2} \quad \text{B1}.$$

Practice 2.2 ■ 1 mark

Use an identity to write $\frac{\sin \theta}{\cos \theta}$ as a single function.

Solution 2.2

Method: Solving a trig equation

Worked steps

$\tan \theta$ **B1**.

Exam-style 3.1 ■ 1 mark

The number of solutions of $\sin \theta = 0.4$ for $0^\circ \leq \theta \leq 360^\circ$ is:

A 0

B 1

C 2

D 4

Solution 3.1

Method: Solving a trig equation

A 0

B 1

C 2



D 4

Worked steps

C. The sine curve gives two solutions in one full turn for $0 < 0.4 < 1$.

Exam-style 3.2 ■ 5 marks

Solve $3 \cos^2 \theta + 2 \sin \theta - 3 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

Solution 3.2

Method: Solving a trig equation

Worked steps

use $\cos^2 \theta = 1 - \sin^2 \theta$: $3(1 - \sin^2 \theta) + 2 \sin \theta - 3 = 0 \Rightarrow -3 \sin^2 \theta + 2 \sin \theta = 0$
M1; $\sin \theta(3 \sin \theta - 2) \cdot (-1) = 0 \Rightarrow \sin \theta = 0$ or $\sin \theta = \frac{2}{3}$ **M1** **A1**; $\sin \theta = 0 \Rightarrow$
 $\theta = 0^\circ, 180^\circ, 360^\circ$; $\sin \theta = \frac{2}{3} \Rightarrow \theta = 41.8^\circ, 138.2^\circ$ **A1** **A1**.

Exam-style 3.3 ■ 4 marks

Solve $2 \sin 2x = 1$ for $0^\circ \leq x \leq 180^\circ$.

Solution 3.3

Method: Solving a trig equation

Worked steps

$\sin 2x = \frac{1}{2}$ **M1**; for $0^\circ \leq 2x \leq 360^\circ$, $2x = 30^\circ, 150^\circ$ **M1**; $x = 15^\circ, 75^\circ$ **A1 A1**.