Exercise walk-through

Circular measure ▪ 1.4

eXercise

You must be able to

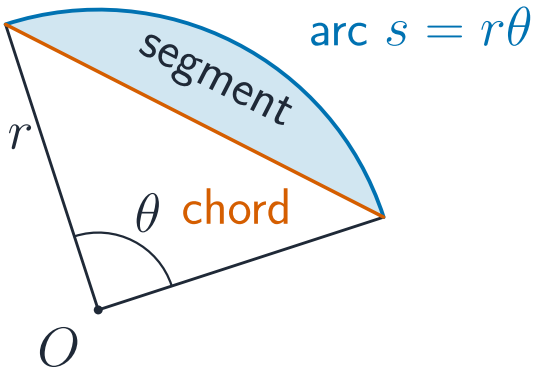
- convert between degrees and radians
- use $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ (with θ in radians)
- find a segment area as sector minus triangle

Method — Sector and segment

For a sector (angle θ in radians): $s = r\theta$, $A = \frac{1}{2}r^2\theta$. Segment $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$.

A sector has $r = 6$**,** $\theta = \frac{2}{3}\pi$**.** Arc $= 6 \cdot \frac{2}{3}\pi = 4\pi$; area $= \frac{1}{2}(36)\frac{2}{3}\pi = 12\pi$.

Method — Sector and segment



A sector with radius r, angle theta, an arc, a chord, and the shaded segment

Practice 2.1 ▪ 1 mark

Convert 120° to radians (exact).

Solution 2.1

Worked steps

$120 \times \dfrac{\pi}{180} = \dfrac{2\pi}{3}$ B1.

Practice 2.2 ▪ 1 mark

A sector has $r = 5$ cm and $\theta = 0.8$ rad. Find its arc length.

Solution 2.2

Method: Sector and segment

Worked steps

$s = r\theta = 5 \times 0.8 = 4$ cm B1.

Exam-style 3.1 ▪ 1 mark

The area of a sector with $r = 4$ and $\theta = \frac{\pi}{6}$ is:

A $\frac{2\pi}{3}$

B $\frac{4\pi}{3}$

C $\frac{\pi}{3}$

D 2π

Solution 3.1

Method: Sector and segment

A $\frac{2\pi}{3}$

B $\frac{4\pi}{3}$ ✓

C $\frac{\pi}{3}$

D 2π

Worked steps

B. $A = \frac{1}{2}(16)\frac{\pi}{6} = \frac{4\pi}{3}$.

Exam-style 3.2 ▪ 3 marks

A sector OAB has centre O, radius 10 cm and angle $AOB = 1.2$ radians.
(a) Find the perimeter of the sector.
(b) Find the area of the **segment** between the chord AB and the arc.

Solution 3.2

Method: Sector and segment

Worked steps

(a) arc $= 10 \times 1.2 = 12$; perimeter $= 12 + 10 + 10 = 32$ cm M1 M1 A1 *arc, add two radii.*

(b) segment $= \frac{1}{2}(10^2)(1.2 - \sin 1.2) = 50(1.2 - 0.9320) = 50(0.2680) = 13.4$ cm^2

M1 M1 A1 *formula, evaluate.*

Exam-style 3.3 ▪ 4 marks

A sector has perimeter 20 cm and area 16 cm^2. Show that the radius r satisfies $r^2 - 10r + 16 = 0$, and hence find the two possible radii.

Solution 3.3

Method: Sector and segment

Worked steps

perimeter: $2r + r\theta = 20$; area: $\frac{1}{2}r^2\theta = 16 \Rightarrow r\theta = \frac{32}{r}$ M1. Substitute: $2r + \frac{32}{r} = 20 \Rightarrow 2r^2 - 20r + 32 = 0 \Rightarrow r^2 - 10r + 16 = 0$ M1 A1. Factorise $(r-2)(r-8) = 0 \Rightarrow r = 2$ or $r = 8$ A1.