Past-paper walk-through

Coordinate geometry ▪ 1.3

eXercise

Questions in this set

- 9709/11 N25 Q10 ▪ 7 marks
- 9709/12 N25 Q7 ▪ 11 marks
- 9709/11 J25 Q6 ▪ 9 marks
- 9709/11 J25 Q8 ▪ 8 marks
- 9709/12 J25 Q8 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 10

9709/11 N25 ▪ Q10 ▪ 7 marks

A circle has equation $x^2 + y^2 + 4y - 21 = 0$ and a straight line has equation $2x + y - 8 = 0$. The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection. **[4]**

(b) The circle has centre C and the two points of intersection are denoted by A and B. Find the area of the triangle ABC. **[3]**

Solution 10

(a) Worked steps

Intersections of a line and a circle: **substitute the line into the circle** and solve the resulting quadratic in one variable.

- From the line, $y = 8 - 2x$. Put that into the circle's equation.
- It reduces to $5x^2 - 40x + 75 = 0$, that is $x^2 - 8x + 15 = 0$.
- $(x - 3)(x - 5) = 0$, so $x = 3$ or $x = 5$.
- Back into the **line** (much easier than the circle): $y = 8 - 6 = 2$ and $y = 8 - 10 = -2$.

Points: $(\mathbf{3}, \mathbf{2})$ and $(\mathbf{5}, -\mathbf{2})$.

Solution 10

Two habits. **Divide the quadratic through by the common factor** — here 5 — before factorising; $5x^2 - 40x + 75$ is far less inviting than $x^2 - 8x + 15$. And **substitute back into the line**, which is linear, rather than into the circle, which would give two y-values per x and require you to discard one.

Two distinct roots means the line is a **chord**. One repeated root would be a tangent, and no real roots would mean it misses — which is what part (b) of many such questions turns on.

Mark scheme

- Substitute $y = 8 - 2x$: $5x^2 - 40x + 75 = 0 \Rightarrow x = 3, 5$. Points $(3, 2)$ and $(5, -2)$.

Solution 10

(b) Worked steps

The triangle ABC has the two intersection points and the **centre** as its vertices, and CA and CB are both **radii**.

- Read the centre and radius from the circle's equation: $C(0, -2)$, $r = 5$.
- $CA = CB = 5$.
- AB: from $(3, 2)$ to $(5, -2)$, length $\sqrt{2^2 + 4^2} = \sqrt{20}$.

The quickest route uses the **perpendicular from the centre to a chord, which bisects it**:

Solution 10

- Half the chord is $\frac{\sqrt{20}}{2} = \sqrt{5}$, so the perpendicular height is $\sqrt{25-5} = \sqrt{20}$.
- Area $= \frac{1}{2} \times \sqrt{20} \times \sqrt{20} = \mathbf{10}$

The two radii and the chord make the triangle **isosceles**, which is what makes that bisection available — and it is the fact worth spotting, because the alternative (the shoelace formula, or $\frac{1}{2}ab\sin C$ after finding an angle) is several times the work.

Solution 10

A sense-check: $\frac{1}{2} \times 5 \times 4 = 10$ also works, since the triangle has legs 3-4-5 in disguise.

Mark scheme

- Centre $C(0, -2)$, radius 5. Area of $ABC = \frac{1}{2}\sqrt{20} \times \sqrt{20} = 10$ (or $\frac{1}{2} \times 5 \times 4$).

Question 7

9709/12 N25 ▪ Q7 ▪ 11 marks

The coordinates of the points P and Q are $(1, 1)$ and $(7, 11)$ respectively. The line segment PQ forms a diameter of a circle.

(a) Find the equation of the circle. **[4]**

(b) Find the equation of the tangent to the circle at the point Q. **[3]**

(c) The other point on the circle with x-coordinate 7 is R. Find the coordinates of the point of intersection of the tangent at Q with the tangent at R. **[4]**

Solution 7

(a)

Mark scheme

- Centre $(4, 6)$; radius$^2 = 3^2 + 5^2 = 34$. Circle: $(x-4)^2 + (y-6)^2 = 34$.

Solution 7

(b)

Mark scheme

- Gradient $PQ = \frac{5}{3}$, so tangent gradient $= -\frac{3}{5}$; tangent at Q: $y = -\frac{3}{5}x + \frac{76}{5}$.

Solution 7

(c)

Mark scheme

- $R = (7, 1)$; tangent at R: $y = \frac{3}{5}x - \frac{16}{5}$. Intersection of the two tangents: $\left(\frac{46}{3},\ 6\right)$.

Question 6

9709/11 J25 ▪ Q6 ▪ 9 marks

The equation of a curve is $2x^2 - kxy + 2 = 0$ and the equation of a line is $y = px + 3$, where k and p are constants.

(a) Given that $k = 2$ and $p = 11$, find the coordinates of the points of intersection of the curve and the line. **[4]**

(b) Given instead that $p = 4$, find the set of values of k for which the curve and the line do not intersect. **[5]**

Solution 6

(a) Worked steps

A circle needs two things: its **centre** and its **radius squared**. Get each, then write the standard form.

- The centre is $(4, 6)$.
- The radius is the distance from the centre to the given point on the circle, found by Pythagoras on the horizontal and vertical gaps — here 3 and 5: $r^2 = 3^2 + 5^2 = 34$.
- $(x - 4)^2 + (y - 6)^2 = \mathbf{34}$

Three points:

Solution 6

- **Leave it as r^2.** The equation wants the square, so computing $r = \sqrt{34}$ and then squaring it again is work that only introduces rounding.
- **The signs inside the brackets are opposite to the centre's coordinates**: a centre of $(4, 6)$ gives $(x - 4)$ and $(y - 6)$. This is the same rule as the translation in a completed square, and it is the commonest slip.
- **Do not expand** unless the question asks for the general form. The standard form displays the centre and radius, which is what makes every later part of these questions easy.

Solution 6

Check by putting the given point back in: the two squared gaps should sum to 34.

Mark scheme

- With $k = 2$, $p = 11$: $x^2 - x(11x + 3) + 1 = 0 \Rightarrow 10x^2 + 3x - 1 = 0 \Rightarrow x = \frac{1}{5}$ or $-\frac{1}{2}$. Points $\left(\frac{1}{5}, \frac{26}{5}\right)$ and $\left(-\frac{1}{2}, -\frac{5}{2}\right)$.

Solution 6

(b)

Mark scheme

- With $p = 4$: $(2 - 4k)x^2 - 3kx + 2 = 0$; no intersection $\Rightarrow$ discriminant < 0: $9k^2 + 32k - 16 < 0 \Rightarrow -4 < k < \frac{4}{9}$.

Question 8

9709/11 J25 ▪ Q8 ▪ 8 marks

The circle with equation $x^2 + y^2 - 6x + 10y - 27 = 0$ intersects the line $x = -2$ at the points P and Q. Find the area of the triangle formed by the tangents to the circle at P and Q, and the line $x = -2$. **[8]**

Solution 8

Mark scheme

- Centre $(3, -5)$, $r = \sqrt{61}$. Line $x = -2$ meets the circle at $P(-2, 1)$, $Q(-2, -11)$. The tangents meet at $T\left(-\frac{46}{5}, -5\right)$. Triangle TPQ: base $PQ = 12$, height $= \frac{36}{5}$, area $= \frac{1}{2} \cdot 12 \cdot \frac{36}{5} = \frac{216}{5} = 43.2$.

Question 8

9709/12 J25 ▪ Q8 ▪ 9 marks

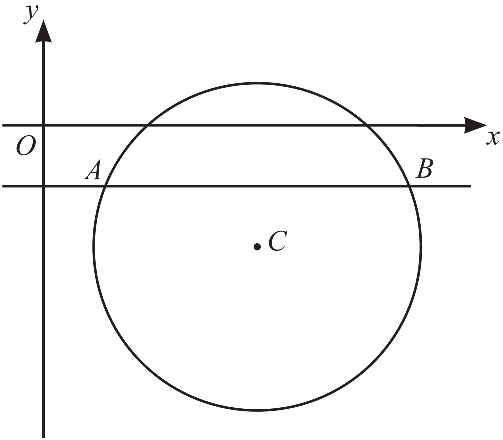
The diagram shows the circle with equation $x^2 + y^2 - 14x + 8y + 36 = 0$ and the line $y = -2$. The line intersects the circle at the points A and B. The centre of the circle is C.

(a) Find the coordinates of A, B and C. **[3]**

(b) Find the angle ACB in radians. Give your answer correct to 3 significant figures. **[2]**

(c) The chord AB divides the circle into two segments. Find the area of the larger segment. **[4]**

Question 8



Solution 8

(a)

Mark scheme

- Centre $C(7,-4)$. Putting $y=-2$ into $x^2+y^2-14x+8y+36=0$ gives $A(2,-2)$ and $B(12,-2)$.

Solution 8

(b)

Mark scheme

- Radius $= \sqrt{29}$. With $\tan\left(\frac{\theta}{2}\right) = \frac{5}{2}$ (or the cosine rule), angle $ACB = 2.38$ rad (AWRT).

Solution 8

(c)

Mark scheme

- Larger sector $= \frac{1}{2} \cdot 29 \cdot (2\pi - \theta) \approx 56.59$; triangle $= \frac{1}{2} \cdot 29 \cdot \sin\theta = 10$. Larger segment area $= 66.6$ (AWRT).