Exercise walk-through

Coordinate geometry ▪ 1.3

eXercise

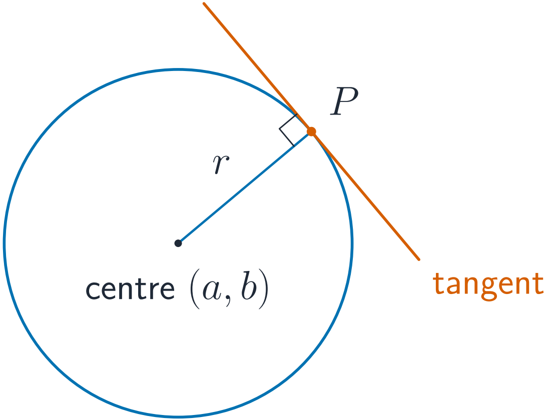
You must be able to

- find the gradient, midpoint and equation of a line
- use the perpendicular-gradient rule ($m_1 m_2 = -1$)
- find a circle's centre and radius; use the tangent–radius right angle

Method — Circle and tangent

$P(1, 1)$ **and** $Q(7, 11)$ **are ends of a diameter. Find the circle's equation.**
Centre $=$ midpoint $= (4, 6)$; radius $= \frac{1}{2}\sqrt{6^2 + 10^2} = \frac{1}{2}\sqrt{136} = \sqrt{34}$. So $(x - 4)^2 + (y - 6)^2 = 34$.
Perpendicular: a line of gradient 2 has perpendicular gradient $-\frac{1}{2}$.

Method — Circle and tangent



A circle with a radius drawn to a point and a tangent meeting it at a right angle

Practice 2.1 ▪ 2 marks

Find the gradient of the line joining $(2, -1)$ and $(6, 7)$.

Solution 2.1

Method: Circle and tangent

Worked steps

$$m = \frac{7-(-1)}{6-2} = \frac{8}{4} = 2 \quad \textbf{M1}\ \textbf{A1}.$$

Practice 2.2 ▪ 2 marks

Write the centre and radius of $(x-3)^2+(y+2)^2=25$.

Solution 2.2

Method: Circle and tangent

Worked steps

centre $(3, -2)$ **B1**; radius 5 **B1**.

Exam-style 3.1 ▪ 1 mark

The line through $(0, 4)$ perpendicular to $y = \frac{1}{3}x + 1$ has gradient:

A $\frac{1}{3}$

B 3

C -3

D $-\frac{1}{3}$

Solution 3.1

Method: Circle and tangent

A $\frac{1}{3}$

B 3

C -3 ✓

D $-\frac{1}{3}$

Worked steps

C. Perpendicular gradient $= -1 \div \frac{1}{3} = -3$.

Exam-style 3.2 ▪ 3 marks

A circle has equation $x^2 + y^2 - 6x + 10y - 2 = 0$.

(a) Find its centre and radius.

(b) Show that the point $(3, 1)$ lies on the circle.

Solution 3.2

Method: Circle and tangent

Worked steps

(a) complete the square: $(x-3)^2-9+(y+5)^2-25-2=0 \Rightarrow (x-3)^2+(y+5)^2=36$ **M1** **A1**; centre $(3,-5)$, radius 6 **A1**.
(b) substitute $(3,1)$: $(3-3)^2+(1+5)^2=0+36=36=6^2$ **M1**; this equals r^2, so $(3,1)$ lies on the circle **A1**.\

Exam-style 3.3 ▪ 4 marks

The points $A(1, 2)$ and $B(5, 10)$ are given.

(a) Find the equation of the perpendicular bisector of AB.

(b) The perpendicular bisector meets the x-axis at C. Find the coordinates of C.

Solution 3.3

Method: Circle and tangent

Worked steps

(a) midpoint $(3, 6)$ **B1**; gradient $AB = \dfrac{10-2}{5-1} = 2$, so bisector gradient $= -\frac{1}{2}$ **M1**; $y - 6 = -\frac{1}{2}(x-3) \Rightarrow y = -\frac{1}{2}x + \frac{15}{2}$ **M1** **A1**.
(b) set $y = 0$: $0 = -\frac{1}{2}x + \frac{15}{2} \Rightarrow x = 15$, so $C(15, 0)$ **M1** **A1**.