Past-paper walk-through

Functions ▪ 1.2

eXercise

Questions in this set

- 9709/11 N25 Q4 ▪ 6 marks
- 9709/11 N25 Q6 ▪ 7 marks
- 9709/12 N25 Q3 ▪ 5 marks
- 9709/12 N25 Q9 ▪ 8 marks
- 9709/11 J25 Q10 ▪ 12 marks
- 9709/12 J25 Q1 ▪ 4 marks
- 9709/12 J25 Q11 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

9709/11 N25 ▪ Q4 ▪ 6 marks

(a) Express $1 - 6x - x^2$ in the form $a - (x + b)^2$, where a and b are constants. **[3]**

(b) The graph of $y = x^2$ is transformed to the graph of $y = 1 - 6x - x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n. **[3]**

Solution 4

(a) Worked steps

The target form is $a - (x + b)^2$, so the x^2 term is **negative** — factor the minus sign out first and complete the square inside.

- $1 - 6x - x^2 = 1 - \left(x^2 + 6x\right)$
- $x^2 + 6x = (x + 3)^2 - 9$
- $1 - \left[(x + 3)^2 - 9\right] = \mathbf{10 - (x + 3)^2}$

So $a = 10$ and $b = 3$.

Solution 4

Two signs to watch, and they are the whole question. Taking the minus out changes $-6x$ to $+6x$ **inside** the bracket, and putting the completed square back in flips the -9 to $+9$. Getting either wrong gives $-8-(x+3)^2$, which is the standard wrong answer. Check by expanding: $10-\left(x^2+6x+9\right)=1-6x-x^2$. ✓ That takes one line and catches both sign errors.

Mark scheme

- $1-6x-x^2=10-(x+3)^2$ (so $a=10$, $b=3$).

Solution 4

(b) Worked steps

Read the transformations **off the completed square**, and remember they are applied to $y = x^2$ in the order stated.

- The **minus sign** in front of the square is a **reflection in the x-axis** — it turns $y = x^2$ into $y = -x^2$.
- The $(x+3)^2$ is a translation of -3 **in the x-direction**: adding inside the bracket shifts **left**.
- The $+10$ is a translation of $+10$ **in the y-direction**.

So the reflection is in the x-**axis**, and $m = \mathbf{-3}$, $n = \mathbf{10}$.

Solution 4

The sign of m is the trap and it is worth understanding rather than memorising: $y = f(x+3)$ takes the value f had at $x+3$ and shows it at x, so the graph moves **left**. Inside the bracket, the shift is opposite to the sign.

The order matters too: reflecting first and then translating is what the question specifies, and it is what the completed square encodes — the $+10$ is applied after the negation.

Mark scheme

- Reflection in the x-axis, followed by a translation with $m = -3$ and $n = 10$.

Question 6

9709/11 N25 ▪ Q6 ▪ 7 marks

Functions f and g are defined by $f(x) = (x + 3)^2 - 12$ for $x \geqslant 0$, and $g(x) = 2x - 5$ for $x \in \mathbb{R}$.

(a) State the range of f. **[1]**

(b) Find an expression for $f^{-1}(x)$. **[2]**

(c) Solve the equation $\mathrm{gf}(x) = 69$. **[4]**

Solution 6

(a)

Mark scheme

- $f(x) \geqslant -3.$

Solution 6

(b)

Mark scheme

- $f^{-1}(x) = -3 + \sqrt{x + 12}$.

Solution 6

(c)

Mark scheme

- $\mathrm{gf}(x) = 2\big((x+3)^2-12\big)-5 = 2(x+3)^2-29 = 69 \Rightarrow 2x^2+12x-80 = 0 \Rightarrow x = 4$ only.

Question 3

9709/12 N25 ▪ Q3 ▪ 5 marks

(a) The graph of $y = f(x)$ is transformed to the graph of $y = f(3x) + 2$. Describe fully the two transformations which have been combined to give the resulting graph. **[3]**

(b) A different graph has equation $y = g(x)$. This graph is stretched by scale factor 3 in the y-direction and then reflected in the y-axis. Write down the equation of the transformed graph in terms of the function g. **[2]**

Solution 3

(a)

Mark scheme

- A stretch with scale factor $\frac{1}{3}$ parallel to the x-axis, and a translation $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ (i.e. 2 units in the y-direction).

Solution 3

(b)

Mark scheme

- $y = 3g(-x)$.

Question 9

9709/12 N25 ▪ Q9 ▪ 8 marks

The function f is defined by $f(x) = \dfrac{4}{(3x-6)^2} + \dfrac{1}{(3x-6)^3}$ for $x > 2$.

(a) Find an expression for $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. **[4]**

(b) State whether f^{-1} exists. Give a reason for your answer. **[1]**

(c) The function g is defined by $g(x) = 4x - 3$ for $x > a$. Find the range of g in terms of the constant a. **[1]**

(d) Find the set of values of a for which the composite function fg exists. **[2]**

Solution 9

(a)

Mark scheme

- $f'(x) = -24(3x-6)^{-3} - 9(3x-6)^{-4}$; for $x > 2$, $(3x-6) > 0$ so both terms are negative — f is a decreasing function.

Solution 9

(b)

Mark scheme

- f^{-1} exists because f is a one-to-one (decreasing) function.

Solution 9

(c)

Mark scheme

- $g(x) > 4a - 3.$

Solution 9

(d)

Mark scheme

- fg requires range$(g) \subseteq$ domain$(f) = (2, \infty)$, so $4a - 3 \geqslant 2 \Rightarrow a \geqslant \frac{5}{4}$.

Question 10

9709/11 J25 ▪ Q10 ▪ 12 marks

The functions f and g are defined by $f(x) = \sqrt{x}$ for $x \geqslant 0$, and $g(x) = 3\sqrt{x+2} - 5$ for $x \geqslant -2$.

(a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. **[5]**

(b) The diagram shows the graph of $y = g(x)$. On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. **[2]**

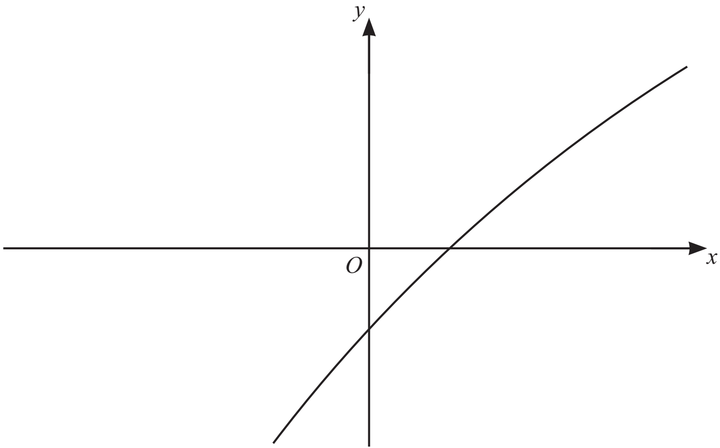
(c) Find an expression for $g^{-1}(x)$. **[2]**

(d) State the range of g^{-1}. **[1]**

(e) The function h is defined by $h(x) = x - 2$ for $x \geqslant 0$. Find the value of $g^{-1}h(4)$. **[1]**

(f) Explain why the composite function hg^{-1} cannot be formed. **[1]**

Question 10



Solution 10

(a)

Mark scheme

- $g(x) = 3f(x+2) - 5$: translate by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, then stretch factor 3 parallel to the y-axis, then translate by $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$.

Solution 10

(b)

Mark scheme

- Reflect $y = g(x)$ in the line $y = x$ (the mirror line $y = x$).

Solution 10

(c)

Mark scheme

- $y = 3\sqrt{x+2} - 5 \Rightarrow g^{-1}(x) = \dfrac{(x+5)^2}{9} - 2$ (for $x \geqslant -5$).

Solution 10

(d)

Mark scheme

- Range of g^{-1}: $g^{-1}(x) \geqslant -2$ (the domain of g).

Solution 10

(e)

Mark scheme

- $h(4) = 2$, so $g^{-1}h(4) = g^{-1}(2) = \frac{49}{9} - 2 = \frac{31}{9}$.

Solution 10

(f)

Mark scheme

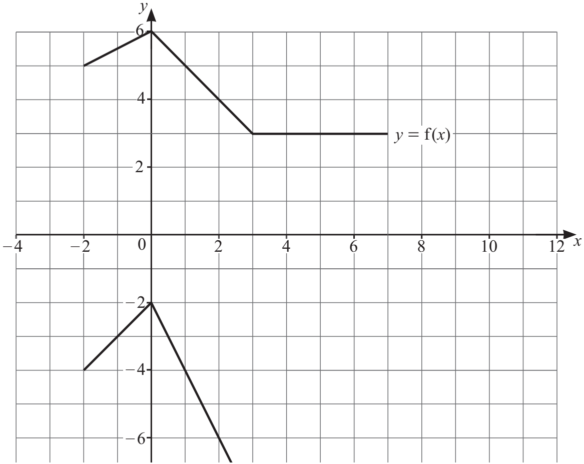
- hg^{-1} needs range$(g^{-1}) \subseteq$ domain(h). Range$(g^{-1}) = [-2, \infty)$ contains negative values, but domain$(h) = [0, \infty)$, so the composition cannot be formed.

Question 1

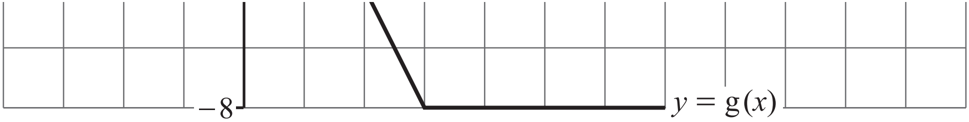
9709/12 J25 ▪ Q1 ▪ 4 marks

The diagram shows the graphs with equations $y = f(x)$ and $y = g(x)$. Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. Make clear the order in which the transformations should be applied. **[4]**

Question 1



Question 1



Solution 1

Mark scheme

- A stretch with factor 2 parallel to the *y*-axis, together with a translation of $\begin{pmatrix} 0 \\ -14 \end{pmatrix}$. (Alternative order: translation $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$ followed by a stretch factor 2 parallel to the *y*-axis.) B2 for 3 correct components, B1 for 2.

Question 11

9709/12 J25 ▪ Q11 ▪ 9 marks

(a) Express $x^2 + 4x + 2$ in the form $(x + a)^2 + b$, where a and b are integers. **[2]**

(b)(i) Find an expression for $f^{-1}(x)$. **[3]**

(b)(ii) Find an expression for $(gf)^{-1}(x)$. **[4]**

Solution 11

(a)

Mark scheme

- $(x+2)^2 - 2$ (i.e. $a = 2,\ b = -2$).

Solution 11

(b)(i)

Mark scheme

- $y = (x+2)^2 - 2 \Rightarrow x = \pm\sqrt{y+2} - 2$; for the domain $x \leqslant -2$ take the negative root: $f^{-1}(x) = -\sqrt{x+2} - 2$.

(b)(ii)

Mark scheme

- $gf(x) = -(x+2)^2 - 2$, so $(gf)^{-1}(x) = -\sqrt{-x-2} - 2$. (Equivalently $(gf)^{-1} = f^{-1}g^{-1}$ with $g^{-1}(x) = -x - 4$.)