

Exercise walk-through

Functions ■ 1.2

eXercise

You must be able to

- find the range of a function and a composite fg
- find an inverse f^{-1} and state its domain
- describe and apply graph transformations

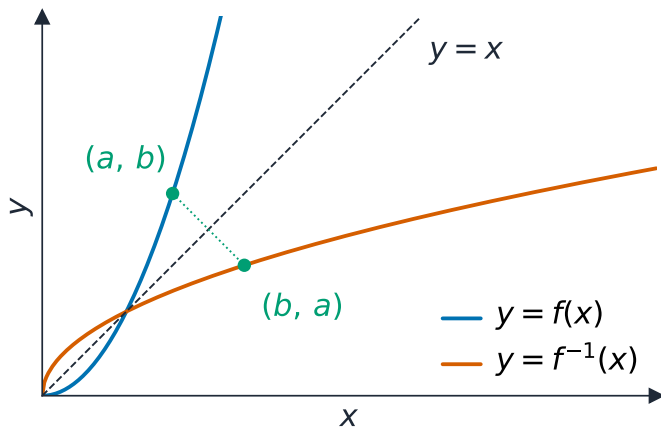
Method — Inverse & composition

$f(x) = (x + 3)^2 - 12$ **for** $x \geq 0$. **Find** $f^{-1}(x)$.

$$y = (x + 3)^2 - 12 \Rightarrow (x + 3)^2 = y + 12 \Rightarrow x = \sqrt{y + 12} - 3.$$

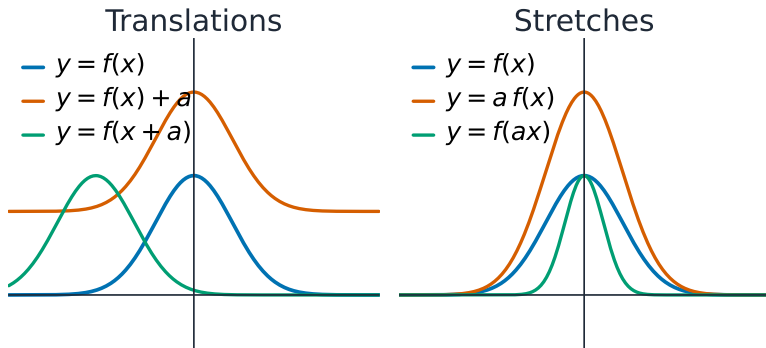
So $f^{-1}(x) = \sqrt{x + 12} - 3$ (positive root, since $x \geq 0$).

Method — Inverse & composition



$y=f(x)$ and its inverse mirror each other across the line $y=x$

Method — Inverse & composition



$f(x) + a$ moves up · $f(x + a)$ moves left · $af(x)$ stretch y · $f(ax)$ compress x if $a > 1$

A curve shifted up and left and stretched

Practice 2.1 ■ 2 marks

$f(x) = 2x - 5$ and $g(x) = x^2$. Find $fg(3)$.

Solution 2.1

Worked steps

$$g(3) = 9 \text{ (M1)}; f(9) = 2(9) - 5 = 13 \text{ (A1)}.$$

Practice 2.2 ■ 1 mark

Describe the single transformation mapping $y = f(x)$ to $y = f(x) - 4$.

Solution 2.2

Worked steps

a translation **down** by 4 **B1**.

Exam-style 3.1 ■ 1 mark

$f(x) = 3 - 2x$. Then $f^{-1}(x)$ is:

A $\frac{3 - x}{2}$

B $\frac{x - 3}{2}$

C $2x - 3$

D $\frac{3 + x}{2}$

Solution 3.1

A $\frac{3-x}{2}$



B $\frac{x-3}{2}$

C $2x-3$

D $\frac{3+x}{2}$

Worked steps

A. $y = 3 - 2x \Rightarrow x = \frac{3-y}{2}$, so $f^{-1}(x) = \frac{3-x}{2}$.

Exam-style 3.2 ■ 3 marks

The function f is defined by $f(x) = x^2 - 6x + 5$ for $x \geq 3$.

- (a) Express $f(x)$ in completed-square form and state the range of f .
- (b) Find $f^{-1}(x)$ and state its domain.

Solution 3.2

Worked steps

(a) $x^2 - 6x + 5 = (x - 3)^2 - 4$ **M1** **A1**; for $x \geq 3$, range is $f(x) \geq -4$ **A1**.

(b) $y = (x - 3)^2 - 4 \Rightarrow x = 3 + \sqrt{x + 4} \rightarrow f^{-1}(x) = 3 + \sqrt{x + 4}$ **M1** **A1** *rearrange*;
domain $x \geq -4$ **A1**.

Exam-style 3.3 ■ 2 marks

The graph of $y = f(x)$ is transformed to $y = 2f(x + 1)$. Describe the **two** transformations, in the correct order.

Solution 3.3

Method: Inverse & composition

Worked steps

translation left by 1 ($x \rightarrow x + 1$) **B1**; then a stretch in the y -direction, scale factor 2 **B1**.