Past-paper walk-through

Quadratics ▪ 1.1

eXercise

Questions in this set

- 9709/11 N25 Q1 ▪ 4 marks
- 9709/11 N25 Q4 ▪ 6 marks
- 9709/12 N25 Q1 ▪ 5 marks
- 9709/12 J25 Q2 ▪ 4 marks
- 9709/12 J25 Q11 ▪ 9 marks

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

9709/11 N25 ▪ Q1 ▪ 4 marks

Find the set of values of the constant k for which the quadratic equation $3kx^2 + (k+8)x + 3 = 0$ has two distinct real roots. **[4]**

Solution 1

Worked steps

"Two distinct real roots" is the **discriminant** condition $b^2 - 4ac > 0$ — strictly greater, because *distinct* rules out the repeated root.
For $3kx^2 + (k+8)x + 3 = 0$:

- $a = 3k,\ b = k+8,\ c = 3$
- $b^2 - 4ac = (k+8)^2 - 36k = k^2 + 16k + 64 - 36k = k^2 - 20k + 64$
- Require $k^2 - 20k + 64 > 0$: $(k-4)(k-16) > 0$

The coefficient of k^2 is positive, so this parabola is **above** the axis **outside** its roots:

Solution 1

$$\mathbf{k < 4} \quad \text{or} \quad \mathbf{k > 16}$$

Three things decide the marks:

- **Expand** $(k+8)^2$ **in full** — $k^2+16k+64$, not k^2+64. That missing $16k$ is the commonest single error in this topic.
- **Solve the quadratic inequality by its shape**, not by dividing. Positive leading coefficient means *outside* the roots; negative means *between* them.
- **Two separate regions**, joined by “or”. A single interval is the answer to the opposite inequality.

Solution 1

Worth noticing but not required here: $k = 0$ would make the equation linear rather than quadratic, so a fully rigorous answer excludes it — and $k = 0$ already lies in $k < 4$.

Mark scheme

- $b^2 - 4ac = (k+8)^2 - 36k = k^2 - 20k + 64 > 0 \Rightarrow (k-4)(k-16) > 0 \Rightarrow k < 4$ or $k > 16$.

Question 4

9709/11 N25 ▪ Q4 ▪ 6 marks

(a) Express $1 - 6x - x^2$ in the form $a - (x + b)^2$, where a and b are constants. **[3]**

(b) The graph of $y = x^2$ is transformed to the graph of $y = 1 - 6x - x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n. **[3]**

Solution 4

(a) Worked steps

The target form is $a - (x + b)^2$, so the x^2 term is **negative** — factor the minus sign out first and complete the square inside.

- $1 - 6x - x^2 = 1 - \left(x^2 + 6x\right)$
- $x^2 + 6x = (x + 3)^2 - 9$
- $1 - \left[(x + 3)^2 - 9\right] = \mathbf{10 - (x + 3)^2}$

So $a = 10$ and $b = 3$.

Solution 4

Two signs to watch, and they are the whole question. Taking the minus out changes $-6x$ to $+6x$ **inside** the bracket, and putting the completed square back in flips the -9 to $+9$. Getting either wrong gives $-8-(x+3)^2$, which is the standard wrong answer. Check by expanding: $10-\left(x^2+6x+9\right)=1-6x-x^2$. ✓ That takes one line and catches both sign errors.

Mark scheme

- $1-6x-x^2=10-(x+3)^2$ (so $a=10,\ b=3$).

Solution 4

(b) Worked steps

Read the transformations **off the completed square**, and remember they are applied to $y = x^2$ in the order stated.

- The **minus sign** in front of the square is a **reflection in the x-axis** — it turns $y = x^2$ into $y = -x^2$.
- The $(x+3)^2$ is a translation of -3 **in the x-direction**: adding inside the bracket shifts **left**.
- The $+10$ is a translation of $+10$ **in the y-direction**.

So the reflection is in the x-**axis**, and $m = \mathbf{-3}$, $n = \mathbf{10}$.

Solution 4

The sign of m is the trap and it is worth understanding rather than memorising: $y = f(x+3)$ takes the value f had at $x+3$ and shows it at x, so the graph moves **left**. Inside the bracket, the shift is opposite to the sign.

The order matters too: reflecting first and then translating is what the question specifies, and it is what the completed square encodes — the $+10$ is applied after the negation.

Mark scheme

- Reflection in the x-axis, followed by a translation with $m = -3$ and $n = 10$.

Question 1

9709/12 N25 ▪ Q1 ▪ 5 marks

(a) Express $9x^2 - 36x + 8$ in the form $p(x + q)^2 + r$, where p, q and r are constants. **[2]**

(b) Hence find the set of values of the constant k for which the equation $9x^2 - 36x + 8 = k$ has no real roots. **[1]**

(c) Find the exact roots of the equation $9x^2 - 36x + 8 = -15$. **[2]**

Solution 1

(a) Worked steps

The form is $p(x+q)^2+r$, so **factor the leading coefficient out of the first two terms only** — not out of the constant.

- $9x^2-36x+8=9\left(x^2-4x\right)+8$
- $x^2-4x=(x-2)^2-4$
- $9\left[(x-2)^2-4\right]+8=9(x-2)^2-36+8=\mathbf{9(x-2)^2-28}$

So $p=9,\ q=-2,\ r=-28.$

Solution 1

The step that goes wrong is multiplying the -4 back out: it is inside the bracket, so it is multiplied by **9**, giving -36, not -4. That single omission produces $9(x-2)^2+4$. Check by expanding: $9\left(x^2-4x+4\right)-28=9x^2-36x+36-28$. ✓

Mark scheme

- $9x^2-36x+8=9(x-2)^2-28$ (so $p=9,\ q=-2,\ r=-28$).

Solution 1

(b)

Mark scheme

- No real roots when k is below the minimum value: $k < -28$.

Solution 1

(c) Worked steps

With the square completed, solving is two steps and no formula.

- $9(x-2)^2 - 28 = -15$
- $9(x-2)^2 = 13$, so $(x-2)^2 = \dfrac{13}{9}$
- $x - 2 = \pm\dfrac{\sqrt{13}}{3}$
- $x = 2 \pm \dfrac{\sqrt{13}}{3} = \boldsymbol{\dfrac{6 \pm \sqrt{13}}{3}}$

Solution 1

Two things. **Both signs** — a square root gives two values, and dropping the $\pm$ loses half the answer. And **exact**, so keep the surd: $\sqrt{13}$ does not simplify, and a decimal loses the mark.

Solution 1

Using the completed square is much quicker than the quadratic formula here, which is why part (a) came first. The single benefit of completing the square is exactly this: the variable appears **once**, so the equation unwinds by inverse operations.

Mark scheme

- $9(x-2)^2 - 28 = -15 \Rightarrow (x-2)^2 = \frac{13}{9} \Rightarrow x = 2 \pm \dfrac{\sqrt{13}}{3} = \dfrac{6 \pm \sqrt{13}}{3}$.

Question 2

9709/12 J25 ▪ Q2 ▪ 4 marks

Find the coordinates of the points of intersection of the curve and the line with equations $2xy + 5y^2 = 24$ and $2x + y + 4 = 0$. **[4]**

Solution 2

Mark scheme

- Eliminate a variable to get $4y^2 - 4y = 24$ (or $16x^2 + 72x + 56 = 0$). Points of intersection: $(-1, -2)$ and $\left(-\frac{7}{2}, 3\right)$.

Question 11

9709/12 J25 ▪ Q11 ▪ 9 marks

(a) Express $x^2 + 4x + 2$ in the form $(x + a)^2 + b$, where a and b are integers. **[2]**

(b)(i) Find an expression for $f^{-1}(x)$. **[3]**

(b)(ii) Find an expression for $(gf)^{-1}(x)$. **[4]**

Solution 11

(a)

Mark scheme

- $(x+2)^2 - 2$ (i.e. $a = 2,\ b = -2$).

Solution 11

(b)(i)

Mark scheme

- $y = (x+2)^2 - 2 \Rightarrow x = \pm\sqrt{y+2} - 2$; for the domain $x \leqslant -2$ take the negative root: $f^{-1}(x) = -\sqrt{x+2} - 2$.

(b)(ii)

Mark scheme

- $gf(x) = -(x+2)^2 - 2$, so $(gf)^{-1}(x) = -\sqrt{-x-2} - 2$. (Equivalently $(gf)^{-1} = f^{-1}g^{-1}$ with $g^{-1}(x) = -x-4$.)