Exercise walk-through

Quadratics ▪ 1.1

eXercise

You must be able to

- write $ax^2 + bx + c$ in completed-square form $a(x + p)^2 + q$
- use $b^2 - 4ac$ to decide the number of real roots
- solve quadratic equations and inequalities
- solve a linear–quadratic pair by substitution

Method — Completing the square & the discriminant

Write $2x^2 - 12x + 5$ as $a(x+p)^2 + q$.

$$2x^2 - 12x + 5 = 2(x^2 - 6x) + 5 = 2((x-3)^2 - 9) + 5 = 2(x-3)^2 - 13.$$

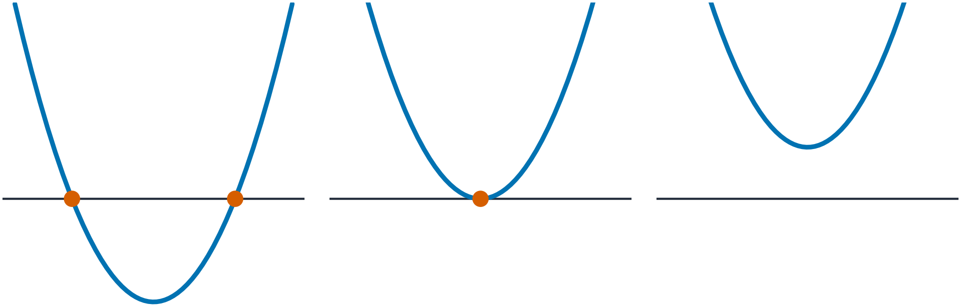
Find the values of k for which $x^2 + kx + 9 = 0$ has two distinct real roots.
Two distinct roots need $b^2 - 4ac > 0$: $k^2 - 36 > 0 \Rightarrow (k-6)(k+6) > 0 \Rightarrow k < -6$ or $k > 6$.

Method — Completing the square & the discriminant

$b^2 - 4ac > 0$
two distinct roots

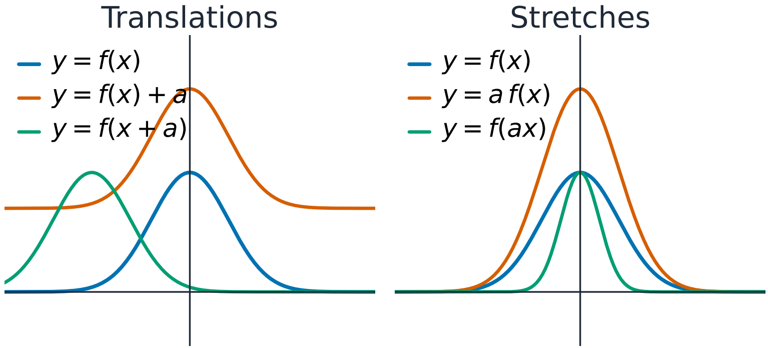
$b^2 - 4ac = 0$
one repeated root

$b^2 - 4ac < 0$
no real roots



Three parabolas: one cutting the x-axis twice, one touching it, one missing it

Method — Completing the square & the discriminant



$f(x) + a$ moves up · $f(x + a)$ moves left · $af(x)$ stretch y · $f(ax)$ compress x if $a > 1$

Completing the square shifts and stretches the parabola

Practice 2.1 ▪ 2 marks

Express $x^2 + 8x + 3$ in the form $(x + p)^2 + q$.

Solution 2.1

Worked steps

$(x+4)^2 - 13$ **M1** **A1** *for* $(x+4)^2$, *for* -13.

Practice 2.2 ▪ 1 mark

State the condition on the discriminant for a quadratic to have **no** real roots.

Solution 2.2

Method: Completing the square & the discriminant

Worked steps

$b^2 - 4ac < 0$ **B1**.

Exam-style 3.1 ▪ 1 mark

The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is:

A 0

B 1

C 2

D 4

Solution 3.1

A 0

B 1

C 2 ✓

D 4

Worked steps

C. Repeated root needs $b^2 - 4ac = 0$: $16 - 8c = 0 \Rightarrow c = 2$.

Exam-style 3.2 ▪ 4 marks

Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$ and hence write down the least value of the expression.

Solution 3.2

Worked steps

$3(x^2+4x)+7=3((x+2)^2-4)+7=3(x+2)^2-5$ **M1** **M1** **A1** *take out 3, complete square*, $=3(x+2)^2-5$; least value $=-5$ (when $x=-2$) **A1**.

Exam-style 3.3 ▪ 4 marks

Find the set of values of k for which the line $y = kx - 2$ meets the curve $y = x^2 - 3x + 2$ at two distinct points.

Solution 3.3

Method: Completing the square & the discriminant

Worked steps

set equal: $x^2 - 3x + 2 = kx - 2 \Rightarrow x^2 - (3+k)x + 4 = 0$ **M1**; two points need $b^2 - 4ac > 0$: $(3+k)^2 - 16 > 0$ **M1**; $(k+7)(k-1) > 0$ **M1**; $k < -7$ or $k > 1$ **A1**.

Exam-style 3.4 ▪ 4 marks

Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$.

Solution 3.4

Worked steps

substitute: $x^2+(2x-1)^2=2 \Rightarrow 5x^2-4x-1=0$ **M1**; $(5x+1)(x-1)=0 \Rightarrow x=-\frac{1}{5}$ or $x=1$ **A1**; $y=2x-1$ gives $(1,1)$ and $\left(-\frac{1}{5},-\frac{7}{5}\right)$ **A1** **A1** *each pair*.