

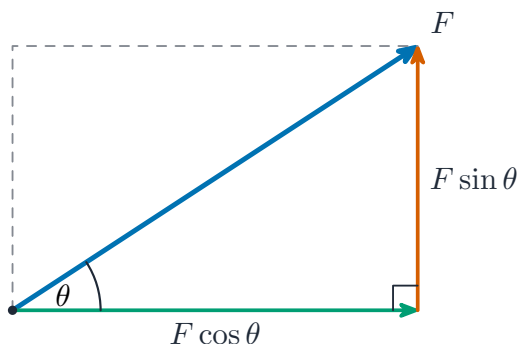
Mechanics

A-Level Mathematics

This handout covers Topic 4: **Mechanics**. It studies how forces make objects move. Throughout this topic, take the acceleration of free fall as $g = 10 \text{ m s}^{-2}$.

Forces and equilibrium

A **force** is a push or a pull. It is a vector, so it has size and direction. Because it is a vector, you can split a force into **components** (usually horizontal and vertical), and you can add several forces into one **resultant**.



A force at angle θ has a horizontal part $F \cos \theta$ and a vertical part $F \sin \theta$.

A particle is in **equilibrium** when the forces are balanced: the vector sum of the forces is zero. In practice this means the components in any direction add to zero.



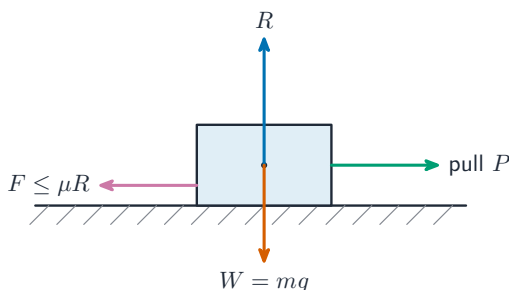
Every mechanics problem is a picture like this. Three forces act on the climber – weight straight down, tension along the rope, and a push from the rock – and because they balance, the climber hangs still in equilibrium. Resolving each into horizontal and vertical parts turns the picture into equations

■ Friction

When two surfaces touch, the **contact force** between them has two parts: the **normal reaction** R , at right angles to the surface, and the **friction** F , along the surface, which opposes sliding. A "smooth" surface is a model with no friction.

Friction can only grow up to a maximum. At that maximum the body is in **limiting equilibrium**, about to slip, and the friction is **limiting friction**. The maximum is set by the **coefficient of friction** μ :

$$F \leq \mu R, \quad \text{with } F = \mu R \text{ at the point of slipping.}$$



On a rough surface the contact force splits into the normal reaction R and friction F (at most μR).

By **Newton's third law**, the two surfaces push on each other with equal and opposite forces.

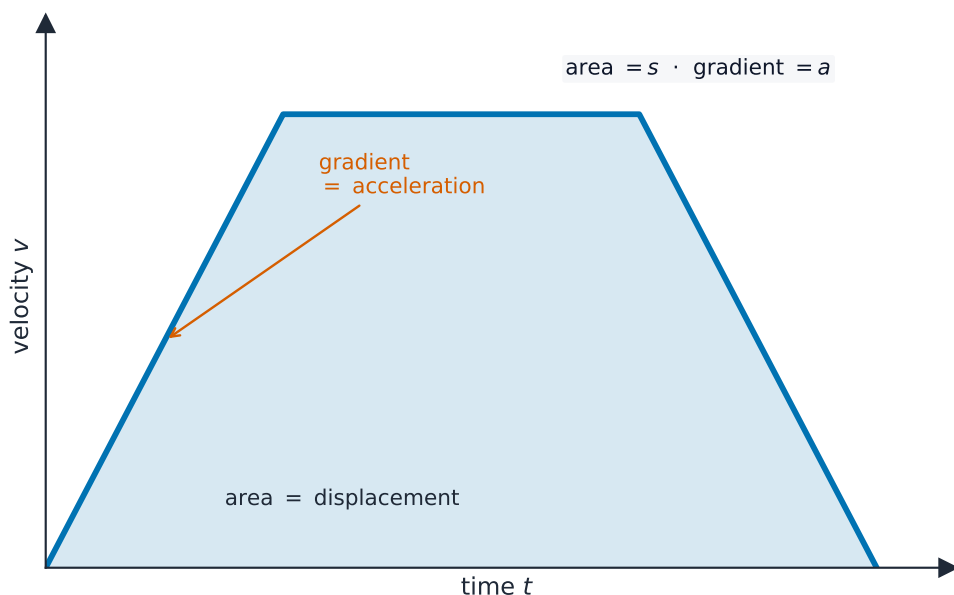
Worked example. A block of weight 20 N rests on a rough horizontal table with coefficient of friction $\mu = 0.4$. Find the largest horizontal force that can be applied before the block slides.

The table's normal reaction balances the weight, so $R = 20$ N. The block is on the point of slipping when friction reaches its maximum $F = \mu R = 0.4 \times 20 = 8$ N. In equilibrium the applied force equals the friction, so the largest force it can resist is 8 N.

Kinematics of motion in a straight line

Distance and **speed** are scalars (size only). **Displacement**, **velocity** and **acceleration** are vectors (size and direction).

On a **velocity-time graph**, the area under the graph is the displacement and the gradient is the acceleration (a negative acceleration is a **deceleration**). On a displacement-time graph, the gradient is the velocity. More generally, differentiate with respect to time to go from displacement to velocity to acceleration, and integrate to go back.



The shaded area gives the distance travelled; the slope of the line gives the acceleration.

For motion with **constant acceleration**, use these formulae (the “suvat” equations):

$$v = u + at, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as, \quad s = \frac{1}{2}(u + v)t.$$

Worked example. A car starts from rest and accelerates at 2.5 m s^{-2} for 4 s. Find its speed and the distance travelled.

$$v = 0 + 2.5 \times 4 = 10 \text{ m s}^{-1}, \quad s = 0 + \frac{1}{2}(2.5)(4^2) = 20 \text{ m}.$$

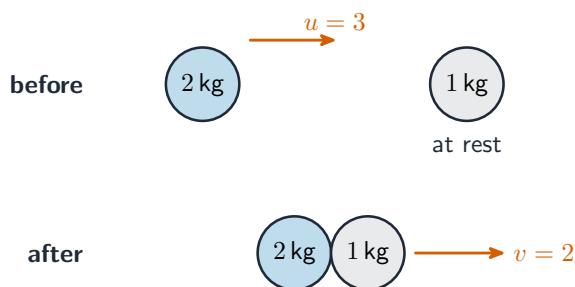
Momentum



A Newton's cradle demonstrates conservation of momentum in collisions.

The **linear momentum** of a body is mass $\times$ velocity. It is a vector. In a direct collision of two bodies, the total momentum is unchanged. This is the **conservation of linear momentum** :

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2.$$



$$2(3) + 1(0) = (2 + 1)v$$

The total momentum is the same before and after the collision.

Worked example. A body of mass 2 kg moving at 3 m s^{-1} hits a stationary body of mass 1 kg, and they stick together. Find their common speed afterwards.

$$2(3) + 1(0) = (2 + 1)v \Rightarrow v = \frac{6}{3} = 2 \text{ m s}^{-1}.$$

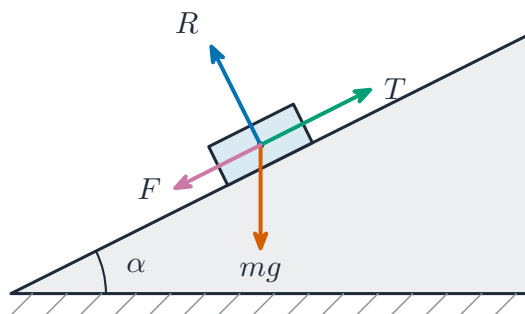
Newton's laws of motion

Newton's laws of motion connect force and acceleration. The key one is: resultant force = **mass** $\times$ acceleration,

$$F = ma.$$

The **weight** of a body is the force of gravity on it: $W = mg$. Forces in a problem may include weight, friction, **tension** in a string, the **thrust** (push) in a rod, and **air resistance**.

For motion on an **inclined plane**, split each force into a part along the slope and a part at right angles to it, then apply $F = ma$ along the slope. For **connected particles** (joined by a string), apply $F = ma$ to each body, or to the whole system.



On a slope, resolve the forces along the slope and at right angles to it.

Worked example. A block of mass 12 kg is pulled up a rough plane by a rope parallel to the slope. The plane is at 20° to the horizontal, the coefficient of friction is 0.4, and the acceleration is 2 m s^{-2} . Find the tension in the rope.

The normal reaction is $R = mg \cos 20^\circ = 120 \cos 20^\circ = 112.8 \text{ N}$, so the friction is $F = \mu R = 0.4 \times 112.8 = 45.1 \text{ N}$. Along the slope, $T - mg \sin 20^\circ - F = ma$:

$$T = ma + mg \sin 20^\circ + F = 12(2) + 120 \sin 20^\circ + 45.1 = 24 + 41.0 + 45.1 = 110 \text{ N (3 s.f.)}.$$

Energy, work and power



A roller coaster trades potential energy for kinetic energy as it rises and falls.

The **work done** by a constant force is the **scalar product** of force and displacement – the force times the distance moved in the direction of the force: $W = Fd \cos \theta$, where θ is the angle between the force and the motion. Work is measured in joules (J).

Energy comes in forms you can calculate:

- **Kinetic energy** (energy of movement): $\text{KE} = \frac{1}{2}mv^2$.
- **Gravitational potential energy** (energy of height): $\text{PE} = mgh$.

The work done by the outside forces equals the change in the total energy. When no friction acts, the total energy stays the same – the **conservation of energy**.

Power is the rate of doing work. For a force pulling in the direction of motion, $P = Fv$ (power = force $\times$ velocity). Power is measured in watts (W).

Worked example. A car engine works at 12 kW while the car moves at 20 m s^{-1} on a level road. Find the driving force.

$$P = Fv \Rightarrow F = \frac{P}{v} = \frac{12000}{20} = 600 \text{ N.}$$

Exam tips

- Draw a clear **force diagram** and resolve into perpendicular components; for equilibrium, each direction sums to zero.
- Use **SUVAT** only for constant acceleration and keep a consistent positive direction.
- Apply $F = ma$ along the direction of motion, including friction ($F = \mu R$) on a rough surface.
- State your **assumptions** (light inextensible string, smooth pulley, particle) – they are often worth a mark.