

6 The Poisson distribution – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English

Write it 3 times (English)

Poisson distribu-
tion

mean

variance

_____	_____	_____
_____	_____	_____
_____	_____	_____

Recall

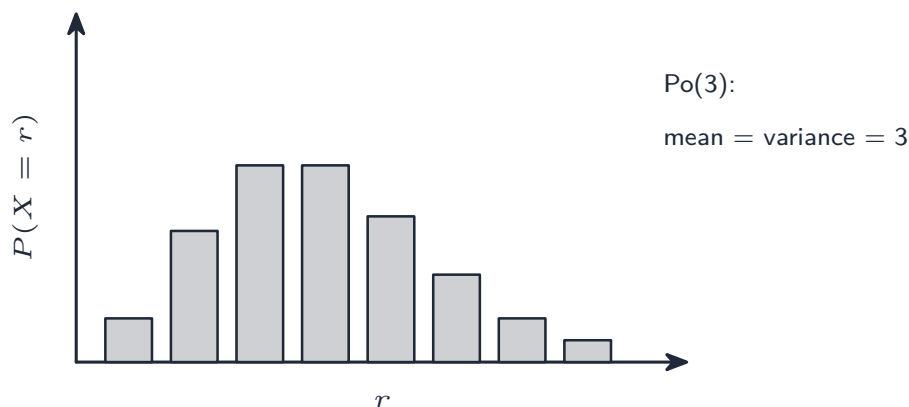
Statistics 2 adds the Poisson model for random events, and rules for combining random variables.

New ideas

Read these first, then do the Practice.

■ 1 The Poisson distribution

The **Poisson distribution** $X \sim \text{Po}(\lambda)$ models the number of random events in a fixed interval, at an average rate λ :



$$P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$$

For a Poisson variable the **mean** and the **variance** both equal λ .

■ 2 Combining random variables

$$E(aX + b) = aE(X) + b$$

$$\text{Var}(aX + b) = a^2 \text{Var}(X)$$

A constant added shifts the mean but not the variance; a multiplier scales the mean by a and the variance by a^2 .

Watch out: for a Poisson variable, *both* the mean and the variance equal λ . When transforming: $E(aX + b) = aE(X) + b$, but $\text{Var}(aX + b) = a^2 \text{Var}(X)$ – the constant b drops out of the variance, and the multiplier is *squared*.

Practice

Try these in order.

6.1 What does the Poisson distribution model? [1]

6.2 For $X \sim \text{Po}(\lambda)$, what is the variance? [1]

6.3 $X \sim \text{Po}(3)$. Use the formula to find $P(X = 2)$. (Take $e^{-3} = 0.0498$.) [2]

6.4 X has mean 5. Find $E(3X - 1)$. [1]

6.5 X has variance 4. Find $\text{Var}(3X - 1)$. [2]

6.6 Challenge. Calls arrive at a help-desk at an average of 4 per hour, modelled by $\text{Po}(4)$. Find the probability of exactly 5 calls in an hour. (Take $e^{-4} = 0.0183$.) [3]