

2 Pure Mathematics 2 – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	Write it 3 times (English)		
logarithm	_____	_____	_____
indices	_____	_____	_____
laws of logarithms	_____	_____	_____
natural logarithm	_____	_____	_____

Recall

In **Part 1** you met the modulus and the factor and remainder theorems. Part 2 adds **logarithms** (the reverse of powers) and the **exponential function**.

New ideas

■ 1 Logarithms

A **logarithm** answers the question "what power?". If $a^x = y$ then $x = \log_a y$. Logarithms and **indices** (powers) are reverse ideas – for example $\log_2 8 = 3$ because $2^3 = 8$.

The **laws of logarithms** let you combine them:

$$\log(mn) = \log m + \log n, \quad \log \frac{m}{n} = \log m - \log n, \quad \log(m^k) = k \log m.$$

$$\log(mn) = \log m + \log n$$

$$\log \frac{m}{n} = \log m - \log n$$

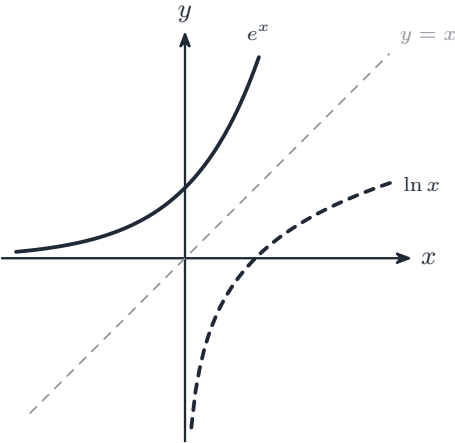
$$\log(m^k) = k \log m$$

Micro-check. Find $\log_3 9$. [1]

■ 2 The exponential function

The **exponential function** e^x and the **natural logarithm** $\ln x$ are **inverse** functions, so $\ln(e^x) = x$ and $e^{\ln x} = x$. Each is the other reflected in the line $y = x$.

When the unknown is **in the power**, take logs of both sides.



Worked example. Solve $2^x = 10$. Take logs: $x \ln 2 = \ln 10$, so $x = \frac{\ln 10}{\ln 2} \approx 3.32$.

Micro-check. Simplify $\ln(e^5)$. [1]

Watch out: a logarithm answers "what power?" – $\log_a y = x$ means $a^x = y$. Combine logs with the laws ($\log mn = \log m + \log n$, etc.). When the unknown is *in the power*, take logs of both sides. e^x and $\ln x$ are inverses.

Practice

2.1 Find (a) $\log_2 16$ and (b) $\log_{10} 1000$. [2]

2.2 Write $\log 6 + \log 5$ as a single logarithm. [1]

2.3 Write $\log(x^3)$ using the power law. [1]

2.4 Solve $3^x = 20$, giving x to 3 significant figures. [2]

2.5 Simplify (a) $\ln(e^7)$ and (b) $e^{\ln 4}$. [2]

2.6 Challenge. Solve $5^{x+1} = 100$, giving x to 3 significant figures. [3]