

1 Pure Mathematics 1 – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	Write it 3 times (English)		
factorising	_____	_____	_____
parabola	_____	_____	_____
quadratic	_____	_____	_____
coefficients	_____	_____	_____
Completing the square	_____	_____	_____
vertex	_____	_____	_____
discriminant	_____	_____	_____
real roots	_____	_____	_____

Recall

At IGCSE you worked with quadratics. Bring this with you:

- You solved quadratic equations by **factorising**.
- You drew the **parabola** – the U-shaped curve of $y = ax^2 + bx + c$.

At AS Pure 1 we add a tidy form that reveals the curve’s turning point, and a quick test for how many solutions an equation has.

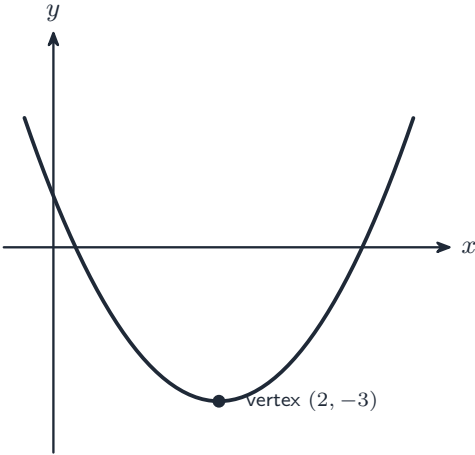
New ideas

■ 1 Completing the square

A **quadratic** has the form $ax^2 + bx + c$; the numbers a, b, c are its **coefficients**.

Completing the square rewrites it as $a(x + p)^2 + q$. This form shows the **vertex** (turning point) of the curve at $(-p, q)$.

Worked example. $x^2 - 4x + 1 = (x - 2)^2 - 4 + 1 = (x - 2)^2 - 3$. The vertex is at $(2, -3)$.

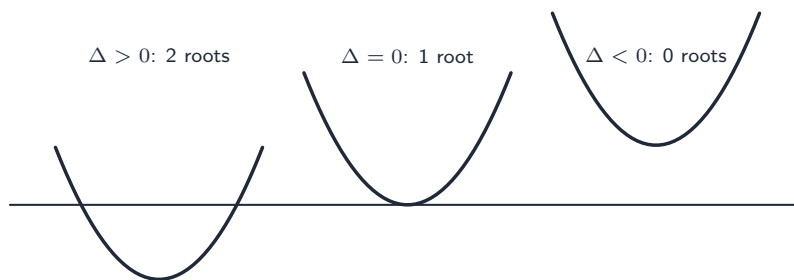


Micro-check. Write $x^2 - 6x$ in the form $(x + p)^2 + q$. [1]

■ 2 The discriminant

The **discriminant** of $ax^2 + bx + c$ is $\Delta = b^2 - 4ac$. It tells you how many **real roots** (real solutions) the equation $ax^2 + bx + c = 0$ has:

- $b^2 - 4ac > 0$ – two real roots;
- $b^2 - 4ac = 0$ – one (repeated) real root;
- $b^2 - 4ac < 0$ – no real roots.



Worked example. For $x^2 - 4x + 1$: $\Delta = (-4)^2 - 4(1)(1) = 16 - 4 = 12 > 0$, so there are **two** real roots.

Micro-check. Find the discriminant of $x^2 + 2x + 5$, and state the number of real roots. [2]

Watch out: to complete the square, halve b and square it. In $a(x+p)^2 + q$ the vertex is at $(-p, q)$ – note the *sign flip* on p . For the discriminant: $b^2 - 4ac > 0 \rightarrow$ two roots, $= 0 \rightarrow$ one, $< 0 \rightarrow$ none.

Practice

1.1 Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$. [2]

1.2 State the vertex of the curve $y = (x - 3)^2 - 4$. [1]

1.3 Find the discriminant of $2x^2 + 3x + 1$, and state how many real roots the equation $2x^2 + 3x + 1 = 0$ has. [2]

1.4 State the number of real roots of $x^2 - 4x + 4 = 0$ (use the discriminant). [2]

1.5 Solve $x^2 - 5x + 6 = 0$ by factorising. [2]

1.6 Challenge. Find the values of k for which $x^2 + kx + 9 = 0$ has exactly one (repeated) real root. [3]