6.5 Hypothesis tests

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS significance level • type ii error • type i error • rejection region • two-tailed

Objective

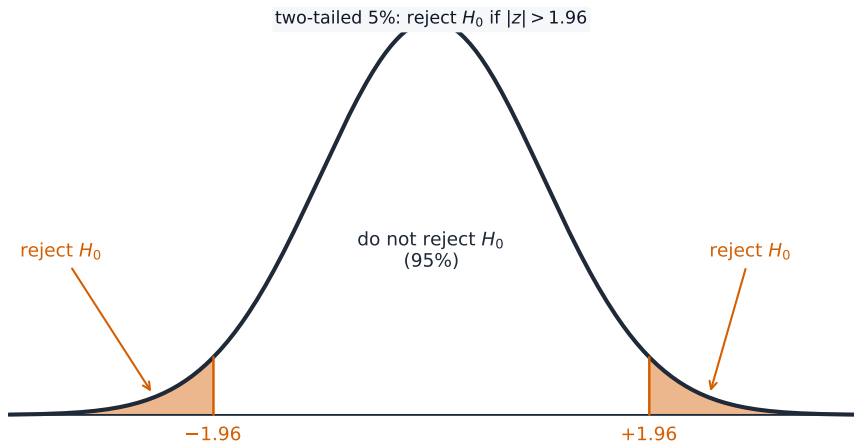
Build the skills to answer exam questions on **hypothesis tests** – stating H_0 and H_1, one- and two-tailed tests, the **test statistic** and **rejection region**, and **Type I/II errors** / .

You must be able to:

- state H_0, H_1 and choose one- or two-tailed
- compute a z test statistic and compare with the critical value
- explain Type I and Type II errors

1 Key knowledge and worked examples

■ A two-tailed test



A two-tailed test at 5% rejects H_0 only if the statistic falls beyond ± 1.96

Claim $\mu = 50$, $\sigma = 8$, $n = 64$, $\bar{x} = 52$, **test at** 5%: $H_0 : \mu = 50$, $H_1 : \mu \neq 50$. $z = \dfrac{52-50}{8/8} = 2$. Since $2 > 1.96$, **reject** H_0.

■ Key knowledge: A one-tailed normal test

State hypotheses about the population parameter, choose the tail from the alternative, and compare the test statistic with the appropriate critical value.

■ Worked example: A one-tailed normal test

Known $\sigma = 4$, $n = 25$ and $\bar{x} = 21.8$. Test $H_0 : \mu = 20$ against $H_1 : \mu > 20$ at 5%. The statistic is $z = (21.8 - 20)/(4/\sqrt{25}) = 2.25$. Since $2.25 > 1.645$, reject H_0: there is evidence that the population mean exceeds 20. This is evidence, not proof.

2 Practice

2.1 State the meaning of a **Type I error**. [1]

2.2 Give the critical z value for a one-tailed test at the 5% level. [1]

3 Exam-style questions

3.1 In a test at the 5% level, H_0 is rejected when the p-value is: [1]

☐ **A** greater than 0.05
B less than 0.05
C equal to 1
D negative

3.2 A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(a) State H_0 and H_1, and whether the test is one- or two-tailed. [2]

(b) Carry out the test and state your conclusion. [4]

3.3 A coin is tossed 20 times to test $H_0 : p = 0.5$ against $H_1 : p > 0.5$ at the 5% level, using the number of heads $X \sim B(20, 0.5)$. The critical region is $X \geqslant 15$.

(a) Find $P(X \geqslant 15)$ under H_0 and explain why $X \geqslant 15$ is used. [3]

(b) State what a Type I error would mean in this context. [1]