6.1 The Poisson distribution

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS poisson distribution • approximation • variance

Objective

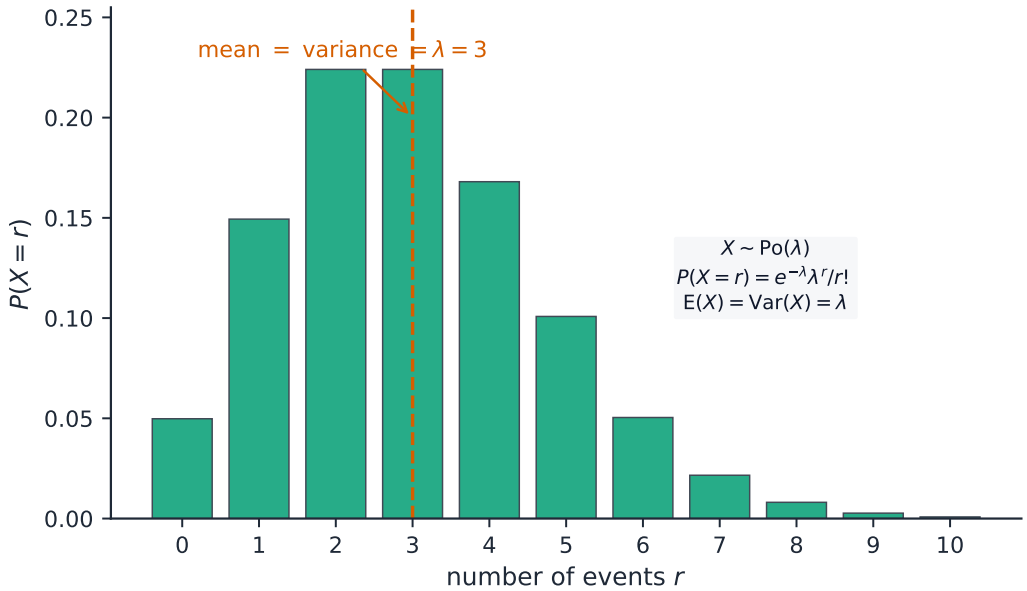
Build the skills to answer exam questions on the **Poisson distribution** – using Po(λ), adding Poisson variables, and the Poisson approximation to the binomial.

You must be able to:

- use $P(X = r) = e^{-\lambda}\dfrac{\lambda^r}{r!}$ (mean = variance = λ)
- combine independent Poisson variables (the sum is Poisson)
- use Po(np) to approximate $B(n, p)$ when n large, p small

1 Key knowledge and worked examples

Poisson probabilities



For a Poisson variable the mean and variance both equal λ

$X \sim \text{Po}(3)$**:** $P(X = 2) = e^{-3}\dfrac{3^2}{2!} = 4.5e^{-3} = 0.224.$

$P(X \geqslant 1) = 1 - P(X = 0) = 1 - e^{-3} = 0.950$.

■ **Key knowledge: Combining Poisson intervals**

A constant-rate Poisson model has mean proportional to interval length. Independent counts over disjoint intervals add.

■ **Worked example: Combining Poisson intervals**

Calls arrive at 3 per hour. Over 20 minutes, $\lambda = 3(20/60) = 1$. Therefore $P(X = 0) = e^{-1} \approx 0.3679$, and $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 1 - 2e^{-1} \approx 0.2642$. The mean 3 would apply to a full hour, not 20 minutes.

2 Practice

2.1 $X \sim \text{Po}(2)$. Find $P(X = 0)$. [2]

2.2 State the variance of Po(4.5). [1]

3 Exam-style questions

3.1 For $X \sim \text{Po}(\lambda)$, the mean equals: [1]

- ☐ **A** λ^2
- ☐ **B** $\sqrt{\lambda}$
- ☐ **C** λ
- ☐ **D** $\dfrac{1}{\lambda}$

3.2 Calls arrive at a switchboard at a mean rate of 1.5 per minute, as a Poisson process.

(a) Find the probability of exactly 2 calls in a given minute. [2]

(b) Find the probability of at least 1 call in a 2-minute period. [3]

3.3 A factory makes items with a 0.5% defect rate. A box holds 400 items. Using a Poisson approximation, find the probability that a box contains exactly 3 defective items. [4]