

3.9 Complex numbers

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS complex numbers • argand diagram • argument • loci
• modulus

Objective

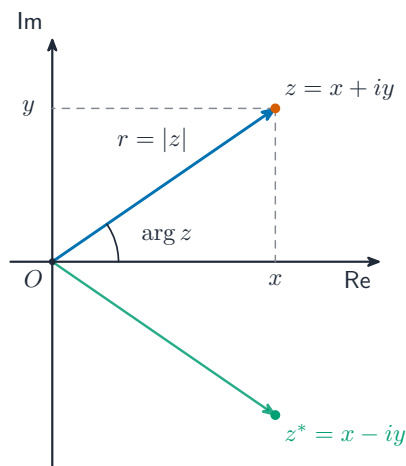
Build the skills to answer exam questions on **complex numbers** – arithmetic and **conjugates**, **modulus** and **argument**, the **Argand diagram**, and **loci**.

You must be able to:

- add, multiply and divide complex numbers (divide using the conjugate)
- find modulus and argument; convert to polar form
- sketch loci such as $|z - a| = r$, $|z - a| = |z - b|$, $\arg(z - a) = \theta$

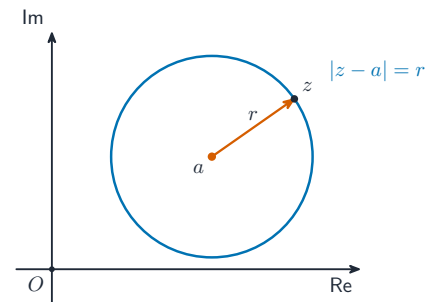
1 Key knowledge and worked examples

■ Division & loci



$|z|$ is the distance from O, $\arg z$ the angle, z^* the reflection in the real axis

$$\frac{3+i}{1-i} = \frac{(3+i)(1+i)}{2} = \frac{2+4i}{2} = 1+2i.$$



$|z - a| = r$ is a circle of radius r centred at a

■ Key knowledge: A modulus locus

The locus $|z - a| = r$ is a circle centred at the point representing a . A locus describes a set of complex numbers, not one solution.

■ Worked example: A modulus locus

For $z = x + iy$, $|z - (2 - i)| = 3$ becomes $|(x - 2) + i(y + 1)| = 3$. Squaring the modulus gives $(x - 2)^2 + (y + 1)^2 = 9$. Thus the centre is $(2, -1)$ and the radius is 3. Check the sign of the imaginary coordinate carefully.

2 Practice

2.1 Find the modulus of $z = 3 + 4i$. [1]

2.2 Write $(2 + i)(3 - i)$ in the form $x + iy$. [2]

3 Exam-style questions

3.1 The argument of $z = -1 + i$ (in radians, $-\pi < \arg z \leq \pi$) is: [1]

☐

- A** $\frac{\pi}{4}$
- B** $\frac{3\pi}{4}$
- C** $-\frac{\pi}{4}$
- D** $\frac{\pi}{2}$

3.2 The complex number z satisfies $z = \frac{10}{2 - i}$.

(a) Express z in the form $x + iy$. [3]

(b) Find $|z|$ and $\arg z$ (in radians). [3]

3.3 On a single Argand diagram, sketch the locus of points z for which:

(a) $|z - (2 + i)| = 2$. [2]

(b) Describe the locus $|z - 3| = |z + 1|$. [2]
