

3.5 Integration (by parts, substitution, partial fractions)

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS substitution • integration • integration by parts
 • partial fractions • rational function

Objective

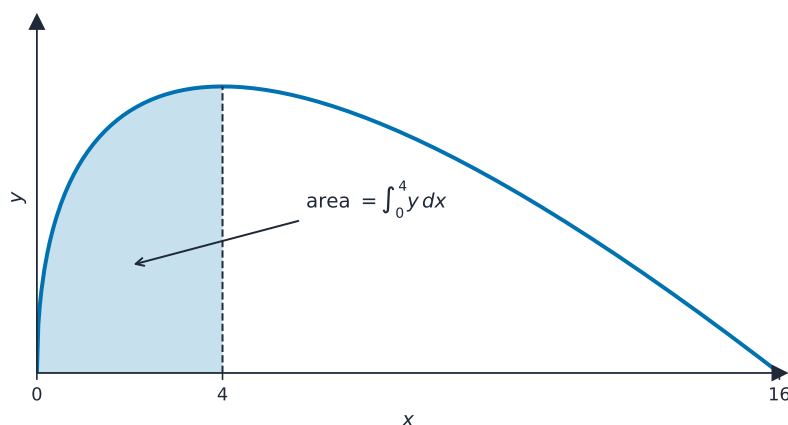
Build the skills to answer exam questions on **integration** in Pure 3 – the standard integral $\frac{1}{x^2+a^2}$, **integration by parts**, **substitution**, partial fractions, and the $\frac{kf'(x)}{f(x)}$ pattern.

You must be able to:

- recognise $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$ and $\int \frac{kf'(x)}{f(x)} dx = k \ln |f(x)| + C$
- integrate by parts and by a given substitution
- integrate a rational function via partial fractions

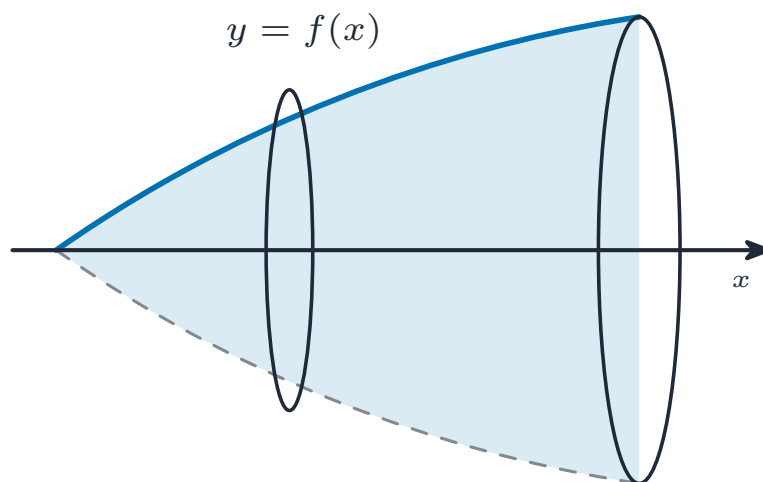
1 Key knowledge and worked examples

■ By parts & the log pattern



$$\int_a^b f(x) dx = \text{net signed area between } y = f(x) \text{ and the } x\text{-axis}$$

A definite integral is still an area; Pure 3 adds the methods to evaluate harder ones



rotate the region around the x -axis to sweep a solid

An application of integration: a volume of revolution

$\int x \cos x \, dx$ – by parts ($u = x$, $dv = \cos x \, dx$): $= x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$.

$\int \frac{2x}{x^2 + 1} \, dx = \ln(x^2 + 1) + C$ (top is the derivative of the bottom).

■ Key knowledge: Definite integration by parts

Choose the factor that simplifies on differentiation as u . Evaluate the boundary term and the remaining integral with the same limits.

■ Worked example: Definite integration by parts

Find $I = \int_0^1 x e^x \, dx$. Take $u = x$ and $dv = e^x \, dx$, so $du = dx$ and $v = e^x$. Then $I = [x e^x]_0^1 - \int_0^1 e^x \, dx = e - (e - 1) = 1$. A derivative check of $(x - 1)e^x$ recovers $x e^x$.

2 Practice

2.1 Find $\int \frac{1}{x^2 + 9} \, dx$.

[2]

2.2 Find $\int \frac{3x^2}{x^3 + 1} dx$. [2]

3 Exam-style questions

3.1 $\int xe^x dx$ equals: [1]

- ☐ **A** $xe^x - e^x + C$
B $xe^x + e^x + C$
C $\frac{1}{2}x^2e^x + C$
D $e^x + C$

3.2 Use the substitution $u = 1 + x^2$ to find $\int_0^2 x\sqrt{1+x^2} dx$. [5]

3.3 (a) Express $\frac{4}{(x)(x+2)}$ in partial fractions. [3]

(b) Hence find $\int_1^3 \frac{4}{x(x+2)} dx$, giving an exact answer. [3]
