# 3.4 Differentiation (inverse tangent, implicit, parametric)

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

**KEY WORDS** quotient • inverse tangent • differentiation

## Objective

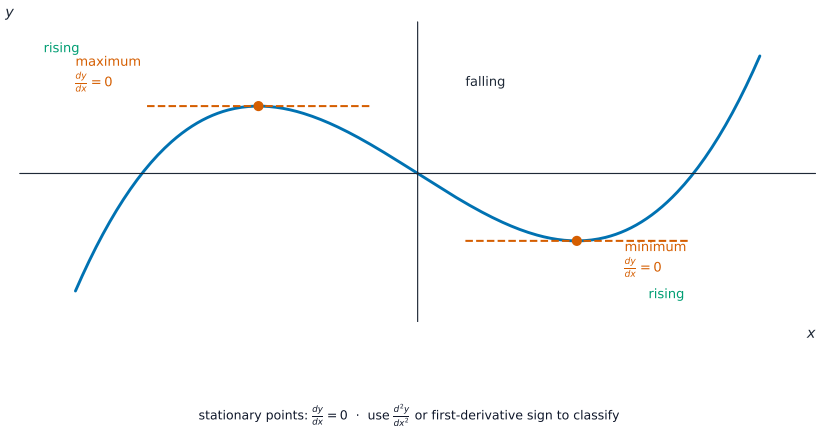
Build the skills to answer exam questions on **differentiation** in Pure 3 – the Pure 2 rules plus the **inverse tangent** derivative, applied to parametric and implicit curves.

**You must be able to:**

- use $\dfrac{d}{dx}\tan^{-1} x = \dfrac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

## 1 Key knowledge and worked examples

### ■ Mixed rules




*Stationary points and gradients still come from differentiating – now with* $\tan^{-1}$ *and implicit methods too*

$y = \tan^{-1}(2x)$**:** chain rule $\to \dfrac{dy}{dx} = \dfrac{1}{1+(2x)^2}\cdot 2 = \dfrac{2}{1+4x^2}$.

**Implicit** $e^{2y} = x + y$**:** $2e^{2y}\dfrac{dy}{dx} = 1 + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2e^{2y}-1}$.

### ■ Key knowledge: An inverse-tangent chain rule

The derivative of $\tan^{-1} u$ is $u\prime/(1+u^2)$. The reciprocal function $1/\tan u$ has a different derivative.

### ■ Worked example: An inverse-tangent chain rule

For $y = \tan^{-1}(2x)$, $dy/dx = 2/(1+4x^2)$. At $x = 1/2$, the point is $(1/2, \pi/4)$ and the gradient is 1. Therefore the tangent is $y - \pi/4 = x - 1/2$. Angles here are in radians.

## 2 Practice

**2.1** Differentiate $y = \tan^{-1}(x^2)$. [2]

**2.2** Differentiate $y = x\ln x$. [2]

## 3 Exam-style questions

**3.1** $\dfrac{d}{dx}\tan^{-1}(3x)$ equals: [1]

☐ **A** $\dfrac{3}{1+9x^2}$

**B** $\dfrac{1}{1+3x^2}$

**C** $\dfrac{3}{1+3x^2}$

**D** $\dfrac{1}{1+9x^2}$

**3.2** A curve has parametric equations $x = 1 + 2\cos t$, $y = 3\sin t$ for $0 \leqslant t \leqslant \pi$. Find $\dfrac{dy}{dx}$ in terms of $t$ and the gradient at $t = \frac{\pi}{6}$. [4]

**3.3** A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$. [3]

(b) Find the gradient at the point $(1, 2)$. [2]