

3.4 Differentiation (inverse tangent, implicit, parametric)

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS quotient • inverse tangent • differentiation

Objective

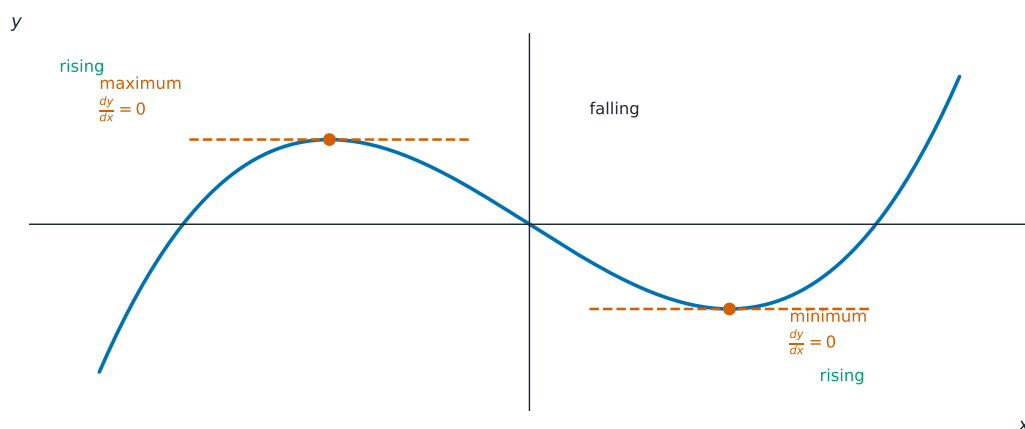
Build the skills to answer exam questions on **differentiation** in Pure 3 – the Pure 2 rules plus the **inverse tangent** derivative, applied to parametric and implicit curves.

You must be able to:

- use $\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$
- apply the product, quotient and chain rules
- differentiate parametric and implicit relations

1 Key knowledge and worked examples

■ Mixed rules



stationary points: $\frac{dy}{dx} = 0$ · use $\frac{d^2y}{dx^2}$ or first-derivative sign to classify

Stationary points and gradients still come from differentiating – now with $\tan^{-1}$ and implicit methods too

$$y = \tan^{-1}(2x): \text{chain rule} \rightarrow \frac{dy}{dx} = \frac{1}{1 + (2x)^2} \cdot 2 = \frac{2}{1 + 4x^2}.$$

Implicit $e^{2y} = x + y$: $2e^{2y} \frac{dy}{dx} = 1 + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2e^{2y} - 1}.$

■ **Key knowledge: An inverse-tangent chain rule**

The derivative of $\tan^{-1} u$ is $u/(1+u^2)$. The reciprocal function $1/\tan u$ has a different derivative.

■ **Worked example: An inverse-tangent chain rule**

For $y = \tan^{-1}(2x)$, $dy/dx = 2/(1 + 4x^2)$. At $x = 1/2$, the point is $(1/2, \pi/4)$ and the gradient is 1. Therefore the tangent is $y - \pi/4 = x - 1/2$. Angles here are in radians.

2 Practice

2.1 Differentiate $y = \tan^{-1}(x^2)$. [2]

2.2 Differentiate $y = x \ln x$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx} \tan^{-1}(3x)$ equals: [1]

- ☐ **A** $\frac{3}{1 + 9x^2}$
- B** $\frac{1}{1 + 3x^2}$
- C** $\frac{3}{1 + 3x^2}$
- D** $\frac{1}{1 + 9x^2}$

3.2 A curve has parametric equations $x = 1 + 2 \cos t$, $y = 3 \sin t$ for $0 \leq t \leq \pi$. Find $\frac{dy}{dx}$ in terms of t and the gradient at $t = \frac{\pi}{6}$. $\frac{dy}{dx}$
[4]

3.3 A curve is given implicitly by $x^2y + y^3 = 10$.

(a) Find $\frac{dy}{dx}$ in terms of x and y . [3]

(b) Find the gradient at the point $(1, 2)$. [2]
