3.3 Trigonometry (identities, R-formula)

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS r-formula • trigonometry

Objective

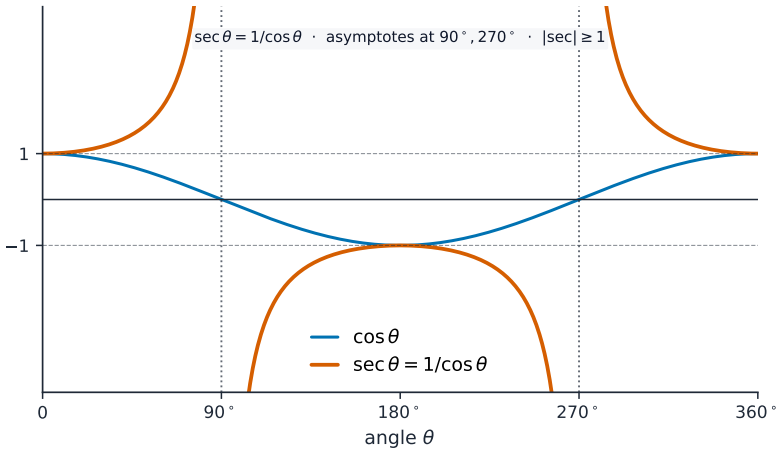
Build the skills to answer exam questions on **trigonometry** in Pure 3 – the reciprocal identities, compound- and double-angle formulae, and the **R-formula** , used in proofs and equations.

You must be able to:

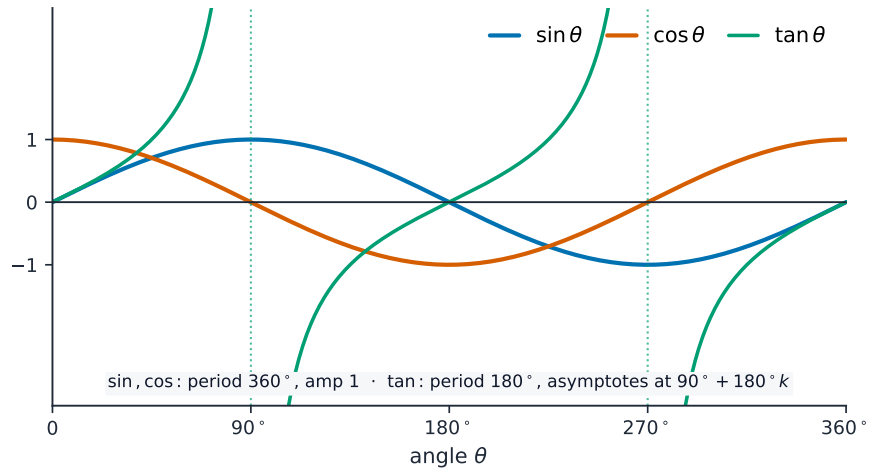
- prove identities using $\sec^2\theta \equiv 1 + \tan^2\theta$ etc.
- expand and use compound/double-angle formulae
- express $a\sin\theta + b\cos\theta$ as $R\sin(\theta \pm \alpha)$ and solve

1 Key knowledge and worked examples

R-formula in an equation



The reciprocal functions and the identities let you reduce an equation to one function



The R-formula rewrites a sum as a single sinusoid

Solve $3\sin\theta + 4\cos\theta = 2$ **for** $0° \leqslant \theta \leqslant 360°$**.** Write LHS as $5\sin(\theta + 53.13°)$: $\sin(\theta + 53.13°) = 0.4$, so $\theta + 53.13° = 23.6°$ or $156.4°$ $(+360°)$. Taking those in range gives $\theta = 103.3°$ or $330.5°$.

Key knowledge: Double-angle equations

Use an identity to put an equation into one trigonometric function, then factor and apply the interval.

Worked example: Double-angle equations

Solve $\cos 2x = \sin x$ for $0 \le x < 2\pi$. Since $\cos 2x = 1 - 2\sin^2 x$, the equation becomes $(2\sin x - 1)(\sin x + 1) = 0$. Thus $x = \pi/6, 5\pi/6, 3\pi/2$. Check each branch in the original equation.

2 Practice

2.1 Write $\cos 2\theta$ in terms of $\cos\theta$ only. [1]

2.2 Find R for $7\sin\theta + 24\cos\theta = R\sin(\theta + \alpha)$. [1]

3 Exam-style questions

3.1 Using $\sin 2\theta = 2\sin\theta\cos\theta$, the value of $\sin 2\theta$ when $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ is: [1]

- **A** $\frac{12}{25}$
- **B** $\frac{24}{25}$
- **C** $\frac{7}{25}$
- **D** $\frac{6}{5}$

3.2 (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Hence solve $4\sin\theta - 3\cos\theta = 2$ for $0° \leqslant \theta \leqslant 360°$. [4]

3.3 Prove the identity $\dfrac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$. [3]