3.2 Logarithmic and exponential functions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS logarithmic and exponential functions

Objective

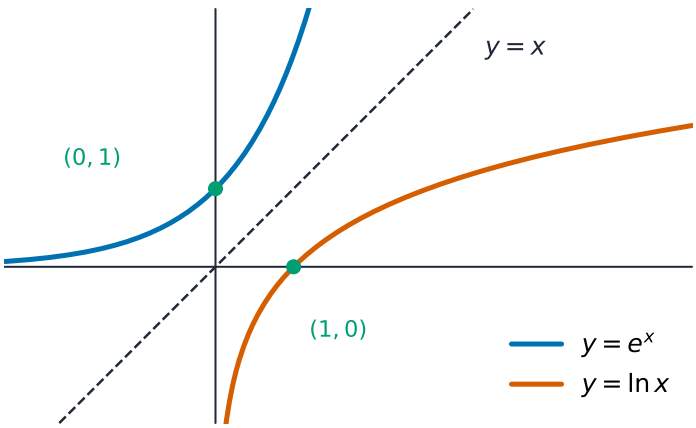
Build the skills to answer exam questions on **logarithmic and exponential functions** in Pure 3 – the same log/exp toolkit as Pure 2, applied to equations and modelling.

You must be able to:

- use the laws of logarithms and $e^x/\ln x$ as inverses
- solve equations with the unknown in the power or inside a log
- handle exponential models (growth/decay)

1 Key knowledge and worked examples

Solving log/exp equations



ln and exp inverse · reflect in $y = x$

e^x and $\ln x$ undo each other – take logs to free an unknown power

Solve $e^{2x} - 5e^x + 6 = 0$**.** Let $u = e^x$: $u^2 - 5u + 6 = 0 \Rightarrow (u-2)(u-3) = 0$. So $e^x = 2$ or 3, giving $x = \ln 2$ or $\ln 3$.

■ **Key knowledge: Exponential decay time**

Separate the initial value from the decay constant. Logarithms turn the fractional remaining amount into a time equation.

■ **Worked example: Exponential decay time**

A mass follows $m = 80e^{-0.2t}$ g. For $m = 20$ g, $m/80 = e^{-0.2t} = 1/4$. Taking logs gives $t = -\ln(1/4)/0.2 = 5\ln 4 \approx 6.93$. This is twice the half-life $\ln 2/0.2$, as two successive halvings leave one quarter.

2 Practice

2.1 Solve $\ln(x-1) = 2$, giving x to 3 s.f. [2]

2.2 Write $2\ln a - \ln b$ as a single logarithm. [1]

3 Exam-style questions

3.1 The solution of $5e^{-x} = 2$ is: [1]

- ☐ **A** $\ln 2.5$
- **B** $-\ln 2.5$
- **C** $\ln 0.4$
- **D** $\ln 10$

3.2 Solve the equation $e^{2x} - 7e^x + 10 = 0$, giving exact answers. [4]

3.3 A quantity decays so that $P = P_0 e^{-kt}$. After 5 years P has fallen to half of P_0.

(a) Find k, to 3 s.f. [3]

(b) Find the time for P to fall to one tenth of P_0. [2]