

3.1 Algebra (partial fractions, rational-power binomial)

Name: _____ Class: _____ Date: _____ Total: 17 marks

KEY WORDS partial fractions • binomial expansion • further algebra
• rational function • converges

Objective

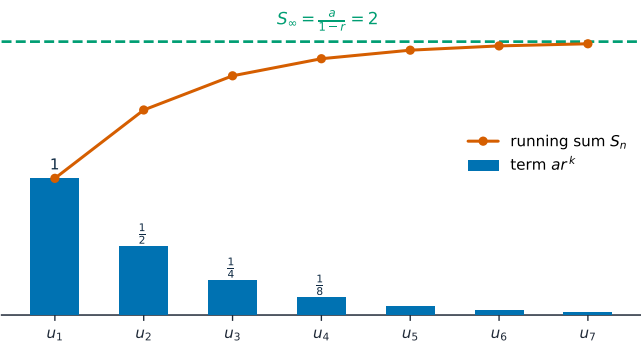
Build the skills to answer exam questions on **further algebra** – **partial fractions** and the **binomial expansion for a rational power**.

You must be able to:

- split a rational function into partial fractions (including a repeated factor)
- expand $(1 + x)^n$ for fractional/negative n (for $|x| < 1$)
- combine the two (expand after splitting)

1 Key knowledge and worked examples

■ Partial fractions & rational-power binomial



The rational-power binomial series converges only for $|x| < 1$ – the terms must shrink

$\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$: $x+4 = A(x-2) + B(x+1)$; $x=2 \Rightarrow B=2$;
 $x=-1 \Rightarrow A=-1$. So $\frac{2}{x-2} - \frac{1}{x+1}$.
 $(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$ for $|x| < 1$.

■ Key knowledge: A repeated denominator factor

A repeated linear factor needs every power up to its multiplicity in the partial-fraction form. Check the result by recombining the fractions.

■ Worked example: A repeated denominator factor

Decompose $(3x+5)/(x+1)^2$. Write $A/(x+1) + B/(x+1)^2$, giving $3x+5 = A(x+1) + B$. Comparing coefficients gives $A=3$ and $B=2$. Recombining gives numerator $3(x+1) + 2 = 3x+5$, so the decomposition is verified.

2 Practice

2.1 Find the first three terms of $(1+x)^{-1}$. [1]

2.2 Write $\frac{5}{(x-1)(x+4)}$ in the form $\frac{A}{x-1} + \frac{B}{x+4}$. [3]

3 Exam-style questions

3.1 The expansion of $(1+2x)^{1/2}$ in ascending powers of x begins: [1]

- ☐ A $1+x - \frac{1}{2}x^2 + \dots$
☐ B $1+x + \frac{1}{2}x^2 + \dots$
☐ C $1+2x - 2x^2 + \dots$
☐ D $1-x + x^2 + \dots$

3.2 Express $\frac{9x+5}{(x+1)(2x-1)}$ in partial fractions. [4]

3.3 (a) Express $\frac{2}{(1-x)(1+2x)}$ in partial fractions. [3]

(b) Hence find the expansion of $\frac{2}{(1-x)(1+2x)}$ up to the term in x^2 . [4]
