

3.1 Algebra (partial fractions, rational-power binomial)

Name: _____ Class: _____ Date: _____

Total: 17 marks

KEY WORDS partial fractions

• binomial expansion

• further algebra

• rational function

• converges

Objective

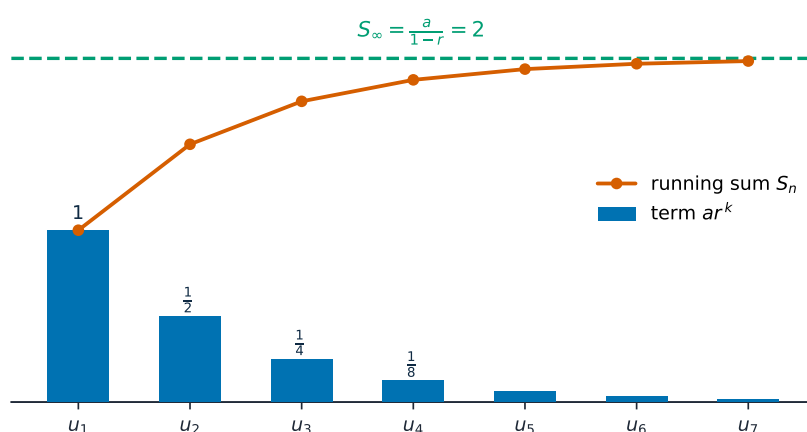
Build the skills to answer exam questions on **further algebra** – **partial fractions**
and the **binomial expansion for a rational power**.

You must be able to:

- split a rational function into partial fractions (including a repeated factor)
- expand $(1+x)^n$ for fractional/negative n (for $|x| < 1$)
- combine the two (expand after splitting)

1 Key knowledge and worked examples

■ Partial fractions & rational-power binomial



$$S_n = \frac{a(1-r^n)}{1-r} \quad \cdot \quad S_\infty = \frac{a}{1-r} \text{ only if } |r| < 1$$

The rational-power binomial series converges only for $|x| < 1$ – the terms must shrink

$$\frac{x+4}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}; \quad x+4 = A(x-2) + B(x+1); \quad x=2 \Rightarrow B=2;$$

$$x=-1 \Rightarrow A=-1. \text{ So } \frac{2}{x-2} - \frac{1}{x+1}.$$

$$(1+x)^{1/2} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots \text{ for } |x| < 1.$$

■ **Key knowledge: A repeated denominator factor**

A repeated linear factor needs every power up to its multiplicity in the partial-fraction form. Check the result by recombining the fractions.

■ **Worked example: A repeated denominator factor**

Decompose $(3x+5)/(x+1)^2$. Write $A/(x+1) + B/(x+1)^2$, giving $3x+5 = A(x+1) + B$. Comparing coefficients gives $A=3$ and $B=2$. Recombining gives numerator $3(x+1) + 2 = 3x+5$, so the decomposition is verified.

2 Practice

2.1 Find the first three terms of $(1+x)^{-1}$. [2]

2.2 Write $\frac{5}{(x-1)(x+4)}$ in the form $\frac{A}{x-1} + \frac{B}{x+4}$. [3]

3 Exam-style questions

3.1 The expansion of $(1+2x)^{1/2}$ in ascending powers of x begins: [1]

- ☐ A $1 + x - \frac{1}{2}x^2 + \dots$
☐ B $1 + x + \frac{1}{2}x^2 + \dots$
☐ C $1 + 2x - 2x^2 + \dots$
☐ D $1 - x + x^2 + \dots$

3.2 Express $\frac{9x+5}{(x+1)(2x-1)}$ in partial fractions. [4]

3.3 (a) Express $\frac{2}{(1-x)(1+2x)}$ in partial fractions. [3]

(b) Hence find the expansion of $\frac{2}{(1-x)(1+2x)}$ up to the term in x^2 . [4]
