# 2.4 Differentiation (product, quotient, parametric, implicit)

Name: _____ Class: _____ Date: _____ **Total: 14 marks**

**KEY WORDS** parametric • parametric equations • product • implicit • quotient rule

## Objective

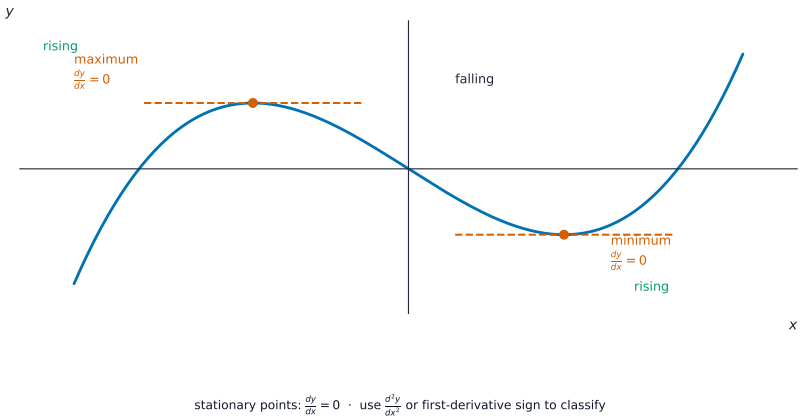
Build the skills to answer exam questions on **differentiation** – standard derivatives ($e^x$, $\ln x$, trig), the **product** and **quotient rules**, and **parametric** and **implicit** differentiation.

**You must be able to:**

- differentiate $e^x$, $\ln x$, $\sin x$, $\cos x$, $\tan x$
- apply the product and quotient rules
- differentiate parametric and implicit relations

## 1 Key knowledge and worked examples

### ■ Product rule & implicit




*A stationary point ($\frac{dy}{dx} = 0$) is still found by differentiating – now with the product/quotient/chain rules*

$y = 6x\cos(x^2+1)$. Product rule ($u = 6x$, $v = \cos(x^2+1)$):

$$\frac{dy}{dx} = 6\cos(x^2+1) - 12x^2\sin(x^2+1).$$

**Implicit:** for $x^2 + y^2 = 25$, differentiate: $2x + 2y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{x}{y}$.

### ■ Key knowledge: A parametric tangent

For parametric equations, $dy/dx = (dy/dt)/(dx/dt)$ when $dx/dt \neq 0$. Both the point and gradient use the same parameter value.

### ■ Worked example: A parametric tangent

Let $x = t^2 + 1$ and $y = t^3$. At $t = 2$, the point is $(5, 8)$. Since $dx/dt = 2t$ and $dy/dt = 3t^2$, $dy/dx = 3t/2$, giving gradient 3. Therefore the tangent is $y - 8 = 3(x - 5)$.

## 2 Practice

**2.1** Differentiate $y = x^2e^x$. [2]

**2.2** Differentiate $y = \dfrac{\sin x}{x}$. [2]

## 3 Exam-style questions

**3.1** $\dfrac{d}{dx}(\ln(3x))$ equals: [1]

☐ **A** $\dfrac{1}{3x}$
**B** $\dfrac{3}{x}$
**C** $\dfrac{1}{x}$
**D** $3\ln x$

**3.2** A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\dfrac{dy}{dx}$ in terms of $t$, and the value of $t$ at the stationary point(s). [4]

**3.3** A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$. [3]

(b) Find the gradient of the curve at the point $(1, 2)$. [2]