2.2 Logarithmic and exponential functions

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS logarithmic and exponential functions
- laws of logarithms
- logarithms
- exponential function
- linear form

Objective

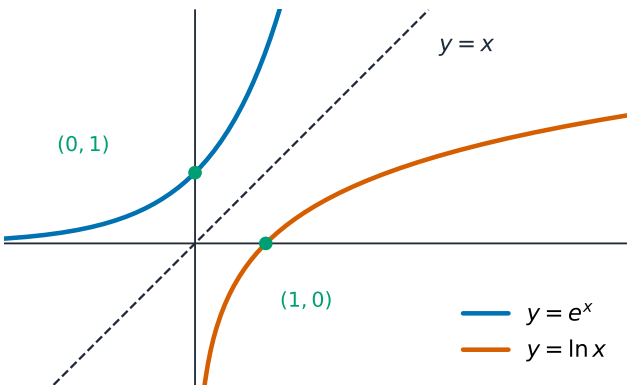
Build the skills to answer exam questions on **logarithmic and exponential functions** – the laws of logs, solving equations with the unknown in the power, and reducing $y = Ax^n$ to **linear form** .

You must be able to:

- use the laws of logarithms and $\ln / e^x$ as inverses
- solve equations such as $a^x = b$ by taking logs
- use a log–log (or log–linear) plot to find unknown constants

1 Key knowledge and worked examples

Solving and linearising



ln and exp inverse · reflect in $y = x$

e^x and $\ln x$ undo each other, so each is the other reflected in $y = x$

Solve $4^x < 0.05$**.** $x \ln 4 < \ln 0.05 \Rightarrow x < \dfrac{\ln 0.05}{\ln 4} = -2.16$ (3 s.f.).

Linear form: $y = Ax^n \Rightarrow \ln y = \ln A + n \ln x$ – plot $\ln y$ against $\ln x$: gradient n, intercept $\ln A$.

■ **Key knowledge: Logarithm domains**

Every real logarithm needs a positive argument. Solve the transformed equation and then check that condition in the original equation.

■ **Worked example: Logarithm domains**

Solve $\ln(x-1) + \ln(x+1) = \ln 8$. The domain is $x > 1$. Combining logs gives $x^2 - 1 = 8$, so $x = \pm 3$. Only $x = 3$ satisfies the domain; $x = -3$ is rejected even though it solves the squared equation.

2 Practice

2.1 Write $\log 12$ in terms of $\log 2$ and $\log 3$. [2]

2.2 Solve $e^{2x} = 7$, giving x to 3 s.f. [2]

3 Exam-style questions

3.1 The solution of $3^x = 20$ is $x =$ [1]

- **A** $\dfrac{\ln 20}{\ln 3}$
- **B** $\dfrac{\ln 3}{\ln 20}$
- **C** $\ln \dfrac{20}{3}$
- **D** $20 - 3$

3.2 Solve the equation $\ln(2x+1) + \ln 3 = \ln(4x+9)$. [3]

3.3 The variables x and y satisfy $y = Ax^n$. When $\ln y$ is plotted against $\ln x$, a straight line of gradient 1.5 passing through (0, 2) is obtained.

(a) Find the values of n and A. [3]

(b) Find y when $x = 4$. [2]