

2.1 Algebra (modulus, factor and remainder theorems)

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS factor • remainder • remainder theorem
 • polynomial • modulus

Objective

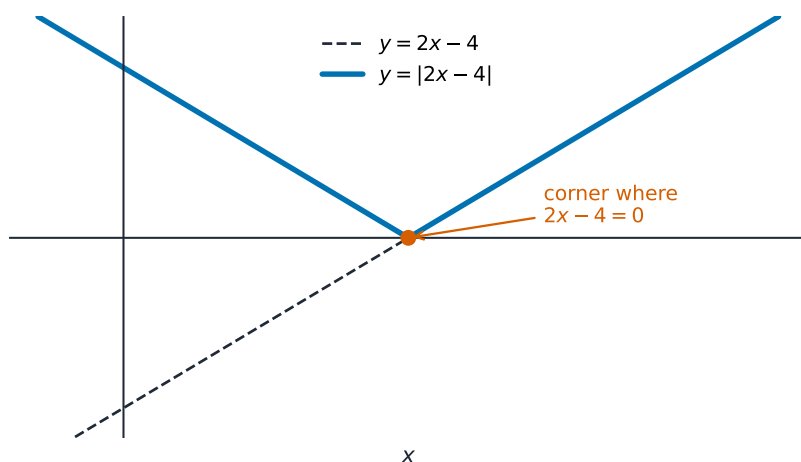
Build the skills to answer exam questions on **algebra** – the **modulus**, and the **factor** and **remainder theorems** for polynomials.

You must be able to:

- solve modulus equations and inequalities ($|x - a| < b$, $|a| = |b| \Leftrightarrow a^2 = b^2$)
- use the remainder theorem (remainder = $p(a)$)
- use the factor theorem to factorise a polynomial

1 Key knowledge and worked examples

■ Modulus & factor theorem



$y = |f(x)|$ reflects parts of $y = f(x)$ below the x-axis above it

Taking the modulus folds the part below the axis up into a “V”

Solve $|3x + 8| < 9$. $-9 < 3x + 8 < 9 \Rightarrow -17 < 3x < 1 \Rightarrow -\frac{17}{3} < x < \frac{1}{3}$.

$p(x) = 2x^4 + kx^3 + kx^2 + 17x + 18$ **has factor** $(x + 2)$. **Find** k . $p(-2) = 0$:
 $32 - 8k + 4k - 34 + 18 = 0 \Rightarrow 16 - 4k = 0 \Rightarrow k = 4$.

■ **Key knowledge: Remainder after division**

Division by $ax + b$ leaves the remainder $p(-b/a)$. Evaluate at the root of the divisor, including its sign.

■ **Worked example: Remainder after division**

For $p(x) = 2x^3 - 3x + 4$, the remainder on division by $2x + 1$ is $p(-1/2)$. Substitution gives $p(-1/2) = 2(-1/2)^3 - 3(-1/2) + 4 = 21/4$. A zero remainder would establish a factor; this non-zero remainder does not.

2 Practice

2.1 Solve $|2x - 1| = 7$. [2]

2.2 Find the remainder when $x^3 - 2x + 5$ is divided by $(x - 2)$. [2]

3 Exam-style questions

3.1 $(x - 3)$ is a factor of $x^3 - 4x^2 + x + c$. The value of c is: [1]

- ☐ **A** -6
B 6
C 12
D -12

3.2 Solve the inequality $|x + 2| > |x - 4|$. [3]

3.3 The polynomial $p(x) = x^3 + ax^2 + bx + 6$ has a factor $(x - 1)$ and leaves a remainder of 8 when divided by $(x + 1)$.

(a) Form two equations in a and b and solve them. [5]

(b) Hence factorise $p(x)$ completely. [3]
