1.5 Trigonometry

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS trigonometric equation • sine • function • identity • trigonometry

Objective

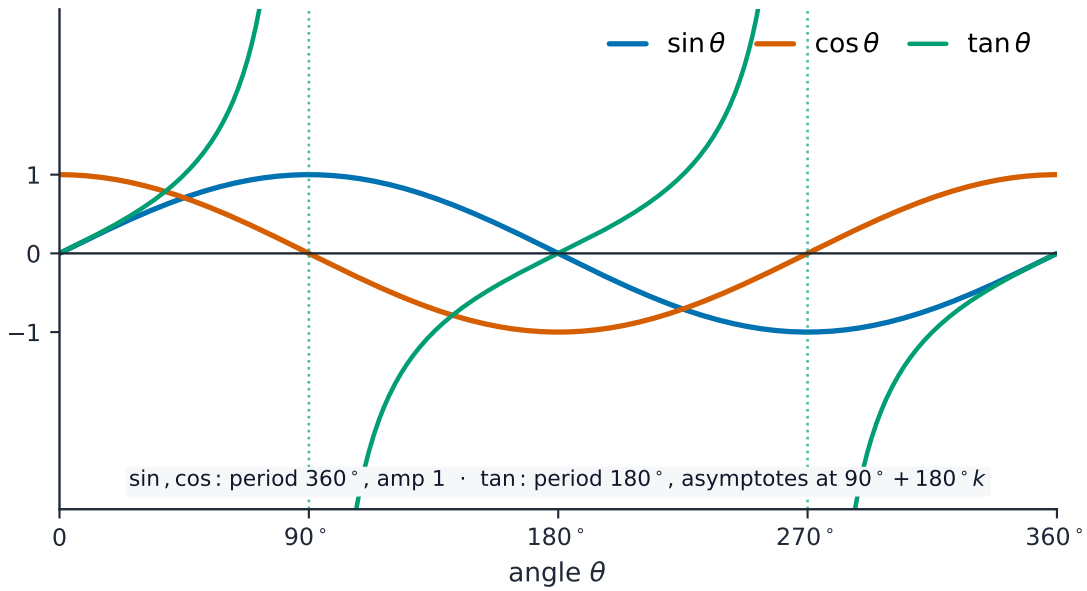
Build the skills to answer exam questions on **trigonometry** – exact values, the **identities**, and solving **trigonometric equations** in a given interval.

You must be able to:

- recall exact values of sin, cos, tan for $30°, 45°, 60°$
- use $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta \equiv 1$
- solve equations (including quadratics in a trig function) over an interval

1 Key knowledge and worked examples

Solving a trig equation



sin *and* cos *stay between* -1 *and* 1*;* tan *shoots off at* $90°$ *and* $270°$

Solve $6\sin\theta = 1 + \dfrac{2}{\sin\theta}$ **for** $-180° < \theta < 180°$**.**

$\times \sin\theta$: $6\sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (3\sin\theta - 2)(2\sin\theta + 1) = 0$. $\sin\theta = \frac{2}{3} \Rightarrow \theta = 41.8°, 138.2°$; $\sin\theta = -\frac{1}{2} \Rightarrow \theta = -30°, -150°$.

■ **Key knowledge: Avoiding lost trigonometric roots**

Factor a trigonometric equation rather than dividing by a trigonometric function that could be zero. List every root in the stated interval.

■ **Worked example: Avoiding lost trigonometric roots**

Solve $2\sin^2 x = \sin x$ for $0 \le x \le 2\pi$. Factor: $\sin x(2\sin x - 1) = 0$. The first branch gives $x = 0, \pi, 2\pi$; the second gives $x = \pi/6, 5\pi/6$. Dividing by $\sin x$ would lose three valid roots.

2 Practice

2.1 Write down the exact value of $\cos 30°$. [1]

2.2 Use an identity to write $\dfrac{\sin\theta}{\cos\theta}$ as a single function. [1]

3 Exam-style questions

3.1 The number of solutions of $\sin\theta = 0.4$ for $0° \leqslant \theta \leqslant 360°$ is: [1]

- ☐ **A** 0
- **B** 1
- **C** 2
- **D** 4

3.2 Solve $3\cos^2\theta + 2\sin\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 360°$. [5]

3.3 Solve $2\sin 2x = 1$ for $0° \leqslant x \leqslant 180°$. [4]