1.1 Quadratics

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS quadratic • quadratic equation • discriminant • simultaneous equations • real roots

Objective

Build the skills to answer exam questions on **quadratics** – completing the square, the **discriminant**, solving equations and inequalities, and simultaneous equations.

You must be able to:

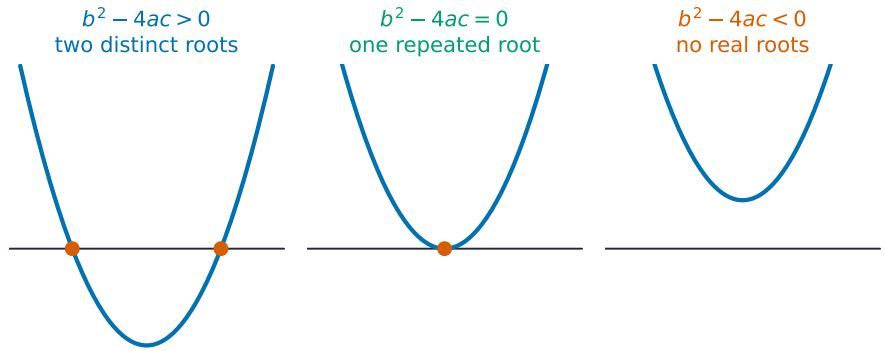
- write $ax^2 + bx + c$ in completed-square form $a(x+p)^2 + q$
- use $b^2 - 4ac$ to decide the number of real roots
- solve quadratic equations and inequalities
- solve a linear–quadratic pair by substitution

1 Key knowledge and worked examples

■ **Completing the square & the discriminant**

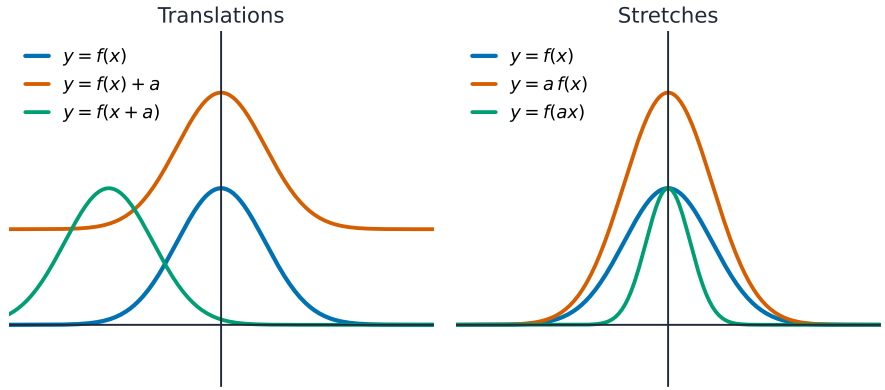
Write $2x^2 - 12x + 5$ as $a(x+p)^2 + q$.

$$2x^2 - 12x + 5 = 2(x^2 - 6x) + 5 = 2((x-3)^2 - 9) + 5 = 2(x-3)^2 - 13.$$



$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (real roots need $b^2 - 4ac \geq 0$)

The sign of $b^2 - 4ac$ decides how many times the parabola meets the x-axis



$f(x) + a$ moves up · $f(x + a)$ moves left · $af(x)$ stretch y · $f(ax)$ compress x if $a > 1$

Completing the square shifts and stretches the parabola

Find the values of k for which $x^2 + kx + 9 = 0$ has two distinct real roots.

Two distinct roots need $b^2 - 4ac > 0$: $k^2 - 36 > 0 \Rightarrow (k-6)(k+6) > 0 \Rightarrow k < -6$ or $k > 6$.

■ **Key knowledge: A quadratic inequality**

For an upward-opening quadratic with two roots, the expression is negative between the roots. Include the roots only for a non-strict inequality.

■ **Worked example: A quadratic inequality**

Solve $x^2 - 5x + 6 \leq 0$. Factor: $f(x) = (x-2)(x-3)$. The roots are $2, 3$ and the leading coefficient is positive. Therefore $f(x) \leq 0$ gives $2 \leq x \leq 3$. Testing $x = 0$ confirms that the outside region is positive.

2 Practice

2.1 Express $x^2 + 8x + 3$ in the form $(x+p)^2 + q$. [2]

2.2 State the condition on the discriminant for a quadratic to have **no** real roots. [1]

3 Exam-style questions

3.1 The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is: [1]

- ☐ **A** 0
- **B** 1
- **C** 2
- **D** 4

3.2 Express $3x^2 + 12x + 7$ in the form $a(x+b)^2 + c$ and hence write down the least value of the expression. [4]

3.3 Find the set of values of k for which the line $y = kx - 2$ meets the curve $y = x^2 - 3x + 2$ at two distinct points. [4]

3.4 Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$. [4]