

5.5 The normal distribution

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS approximation • normal distribution
 • continuity correction • standardisation • mean

Objective

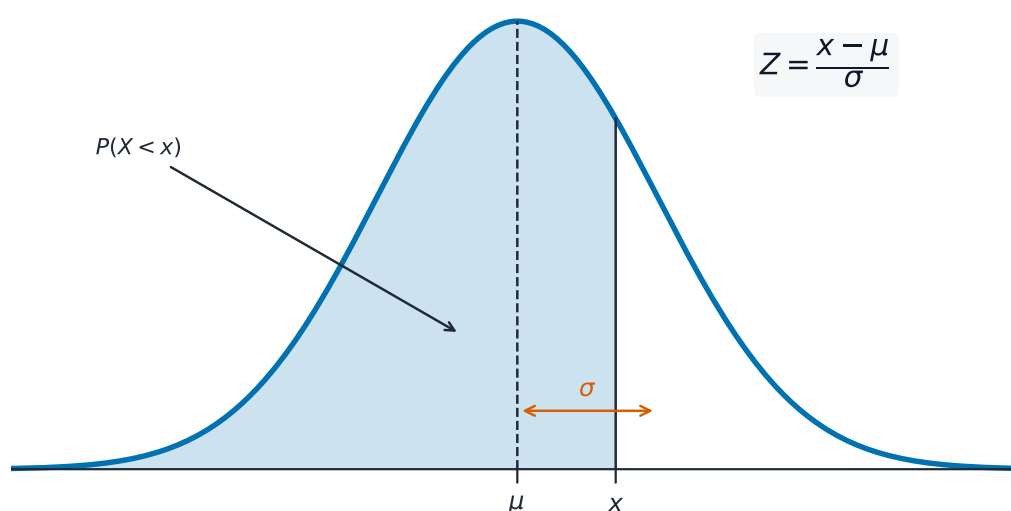
Build the skills to answer exam questions on the **normal distribution** – **standardisation**, reading the Φ table, finding an unknown μ or σ , and the **normal approximation to the binomial**.

You must be able to:

- standardise with $Z = \frac{X - \mu}{\sigma}$ and read Φ
- find an unknown mean or standard deviation from a given probability
- use the normal approximation with a **continuity correction**

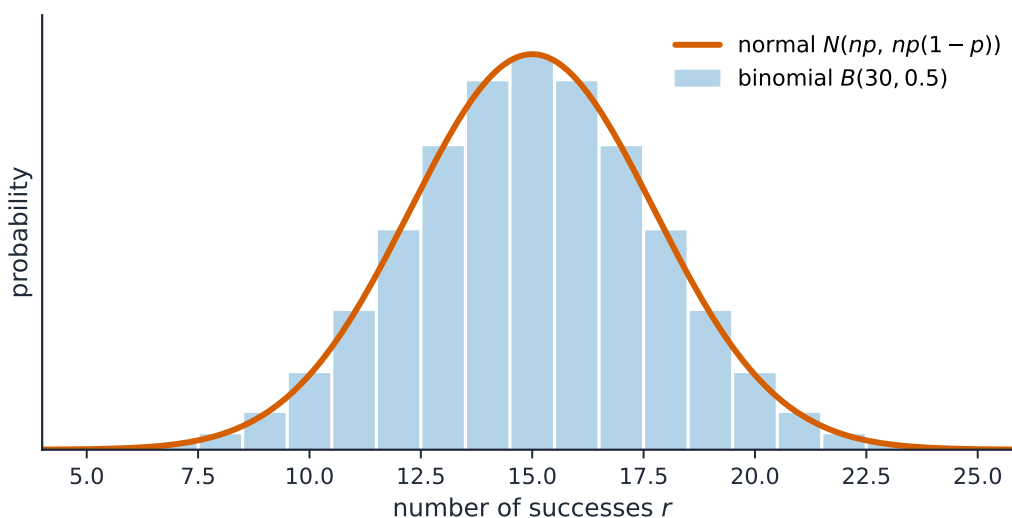
1 Key knowledge and worked examples

■ Standardise & find μ



A normal probability is an area under the bell curve; standardising rescales it to $Z \sim N(0, 1)$

$X \sim N(\mu, 0.14^2)$, $P(X < 1.48) = 0.22$. $\Phi^{-1}(0.22) = -0.772$, so $\frac{1.48 - \mu}{0.14} = -0.772 \Rightarrow \mu = 1.59$.



approx $N(np, np(1-p))$ when n large · continuity correction

For large n the binomial $\approx N(np, np(1-p))$ – add a ± 0.5 continuity correction

■ Key knowledge: Reversing standardisation

For a normal distribution, translate a probability into a z -value before rearranging $z = (x - \mu)/\sigma$.

■ Worked example: Reversing standardisation

Let $X \sim N(50, 8^2)$. Given $P(Z < 1.645) = 0.95$, the 95th percentile satisfies $(x - 50)/8 = 1.645$. Thus $x = 50 + 8(1.645) = 63.16$. This is an upper percentile above the mean; using a minus sign would identify the lower 5th percentile.

2 Practice

2.1 $X \sim N(50, 4^2)$. Find the Z value for $X = 56$. [2]

2.2 State the continuity correction used for $P(X \geq 20)$ in a normal approximation. [1]

3 Exam-style questions

3.1 $X \sim N(0, 1)$. Using $\Phi(1) = 0.8413$, $P(Z > 1)$ is: [1]

☐

A 0.8413

B 0.1587

C 0.3413

D 0.5

3.2 The heights of plants are $X \sim N(82, 5^2)$ cm.

(a) Find $P(X > 90)$. [3]

(b) Find the height exceeded by the tallest 10% of plants. [3]

3.3 A fair coin is tossed 100 times. Using a normal approximation, find the probability of obtaining at least 60 heads. [5]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $Z = \frac{56 - 50}{4} = 1.5$

2.2

- use $P(X \geq 19.5)$

3.1 B

- $P(Z > 1) = 1 - 0.8413 = 0.1587$

3.2

(a)

- $Z = \frac{90 - 82}{5} = 1.6$; $P(X > 90) = 1 - \Phi(1.6) = 1 - 0.9452 = 0.0548$

(b)

- top 10%: $\Phi(z) = 0.90 \Rightarrow z = 1.282$; $x = 82 + 1.282(5) = 88.4$ cm

3.3

- $X \sim B(100, 0.5) \approx N(50, 25)$; continuity correction $P(X \geq 60) \rightarrow P(Y > 59.5)$;
 $Z = \frac{59.5 - 50}{5} = 1.9$; $P = 1 - \Phi(1.9) = 1 - 0.9713 = 0.0287$