

5.3 Probability

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS probability • independent • mutually exclusive
 • conditional probability • independent events

Objective

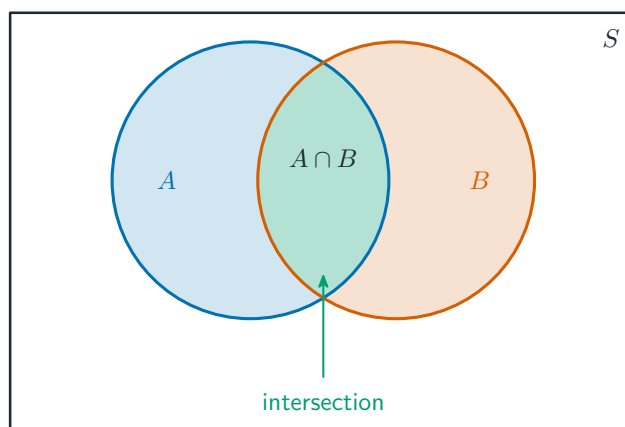
Build the skills to answer exam questions on **probability** – the addition and multiplication rules, **mutually exclusive** and **independent** events, and **conditional probability**.

You must be able to:

- use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- test independence with $P(A \cap B) = P(A)P(B)$
- use $P(A | B) = \frac{P(A \cap B)}{P(B)}$

1 Key knowledge and worked examples

■ Rules & independence



The overlap is $A \cap B$; the addition rule subtracts it once so it is not double-counted

$P(A) = 0.5$, $P(B) = 0.4$, $P(A \cap B) = 0.2$. **Independent?** $P(A)P(B) = 0.2 = P(A \cap B)$ – yes.

Conditional: $P(A | B) = \frac{0.2}{0.4} = 0.5$.

■ **Key knowledge: A conditional probability**

Conditioning changes the sample space. Use $P(A | B) = P(A \cap B)/P(B)$ with $P(B) > 0$.

■ **Worked example: A conditional probability**

Of 40 students, 18 study art, 16 study biology, and 6 study both. Given that a student studies biology, the probability of art is $P(A | B) = 6/16 = 3/8$. By contrast, $P(A \cap B) = 6/40$. The denominator is the conditioned group, not the whole class.

2 Practice

2.1 Events are mutually exclusive with $P(A) = 0.3$, $P(B) = 0.45$. Find $P(A \cup B)$. [2]

2.2 State the test for two events A and B to be independent. [1]

3 Exam-style questions

3.1 For independent events with $P(A) = 0.6$ and $P(B) = 0.5$, $P(A \cap B)$ is: [1]

- ☐ **A** 1.1
B 0.3
C 0.1
D 0

3.2 A and B are events with $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$. [2]

(b) Determine whether A and B are independent. [2]

(c) Find $P(A \mid B)$. [2]

3.3 A bag has 5 red and 3 blue balls. Two are drawn without replacement. Find the probability that both are red. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- mutually exclusive
- $P(A \cup B) = 0.3 + 0.45 = 0.75$

2.2

- $P(A \cap B) = P(A) \times P(B)$

3.1 B

- $0.6 \times 0.5 = 0.3$

3.2

(a)

- $0.8 = 0.6 + 0.5 - P(A \cap B) \Rightarrow P(A \cap B) = 0.3$

(b)

- $P(A)P(B) = 0.6 \times 0.5 = 0.3 = P(A \cap B)$, so they **are** independent

(c)

- $P(A \mid B) = \frac{0.3}{0.5} = 0.6$

3.3

- $P(\text{both red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$