3.7 Vectors

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS vectors • position vector • scalar product • scalar • magnitude

Objective

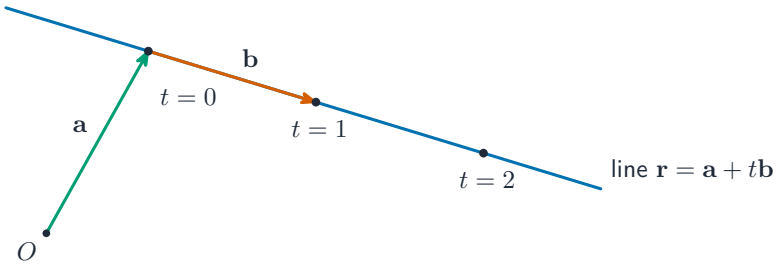
Build the skills to answer exam questions on **vectors** – magnitude and unit vectors, the **vector equation of a line**, the **scalar product**, and the angle between lines.

You must be able to:

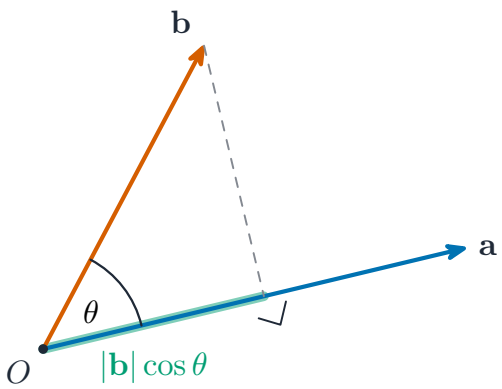
- find magnitude, a unit vector and a displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- write and use $\mathbf{r} = \mathbf{a} + t\mathbf{b}$; test for intersection
- use $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$ to find angles / perpendicularity

1 Key knowledge and worked examples

Line and scalar product



$\mathbf{r} = \mathbf{a} + t\mathbf{b}$: *start at* $\mathbf{a}$, *add* t *copies of the direction* $\mathbf{b}$



$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$*; it is* 0 *when the vectors are perpendicular*

Angle between $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$: $\mathbf{a} \cdot \mathbf{b} = 8$, $|\mathbf{a}| = |\mathbf{b}| = 3$, so $\cos\theta = \frac{8}{9}$, $\theta = 27.3°$.

■ Key knowledge: An intersection of vector lines

Equate corresponding components and use separate parameters for the two lines. A candidate must satisfy all three component equations.

■ Worked example: An intersection of vector lines

Take $\mathbf{r} = (1, 0, 2) + t(1, 2, -1)$ and $\mathbf{r} = (3, 4, 0) + s(0, 1, 1)$. The x-component gives $t = 2$, and the y-component then gives $s = 0$. The z-components both give 0, so the intersection is $(3, 4, 0)$. Failure of the last equation would mean there is no intersection.

2 Practice

2.1 Find $|\mathbf{v}|$ for $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$. [2]

2.2 Find $\mathbf{a} \cdot \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$. [2]

3 Exam-style questions

3.1 Two vectors are perpendicular when their scalar product is: [1]

- ☐ **A** 1
- **B** 0
- **C** the product of their magnitudes
- **D** undefined

3.2 The line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the position vector of the point on l where $t = -3$. [2]

(b) Find the acute angle between l and the direction $\mathbf{d} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$. [4]

3.3 Points $A(1, 2, 1)$, $B(3, 3, 4)$ and $C(2, 0, 3)$ are given.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2]

(b) Find angle BAC. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $|\mathbf{v}| = \sqrt{9+16+144} = \sqrt{169} = 13$

2.2

- $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 1 = 0$

3.1 B

- Perpendicular vectors have scalar product 0

3.2

(a)

- $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ -7 \\ 9 \end{pmatrix}$

(b)

- direction of l is $\mathbf{b} = (1, 2, -2)$; $\mathbf{b} \cdot \mathbf{d} = (1)(2) + (2)(-1) + (-2)(2) = 2 - 2 - 4 = -4$; $|\mathbf{b}| = \sqrt{1+4+4} = 3$ and $|\mathbf{d}| = \sqrt{4+1+4} = 3$; $\cos\theta = \dfrac{-4}{9}$, so $\theta = 116.4°$ – obtuse. The **acute** angle between the two directions is therefore $180° - 116.4° = 63.6°$; equivalently, take the modulus first: $\cos\theta = \frac{4}{9}$

3.3

(a)

- $\overrightarrow{AB} = (2, 1, 3)$, $\overrightarrow{AC} = (1, -2, 2)$

(b)

- $\overrightarrow{AB} \cdot \overrightarrow{AC} = 2 - 2 + 6 = 6$; $|\overrightarrow{AB}| = \sqrt{14}$, $|\overrightarrow{AC}| = 3$; $\cos(BAC) = \dfrac{6}{3\sqrt{14}} = 0.5345 \Rightarrow BAC = 57.7°$