# 3.3 Trigonometry (identities, R-formula)

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

**KEY WORDS** r-formula • trigonometry

## Objective

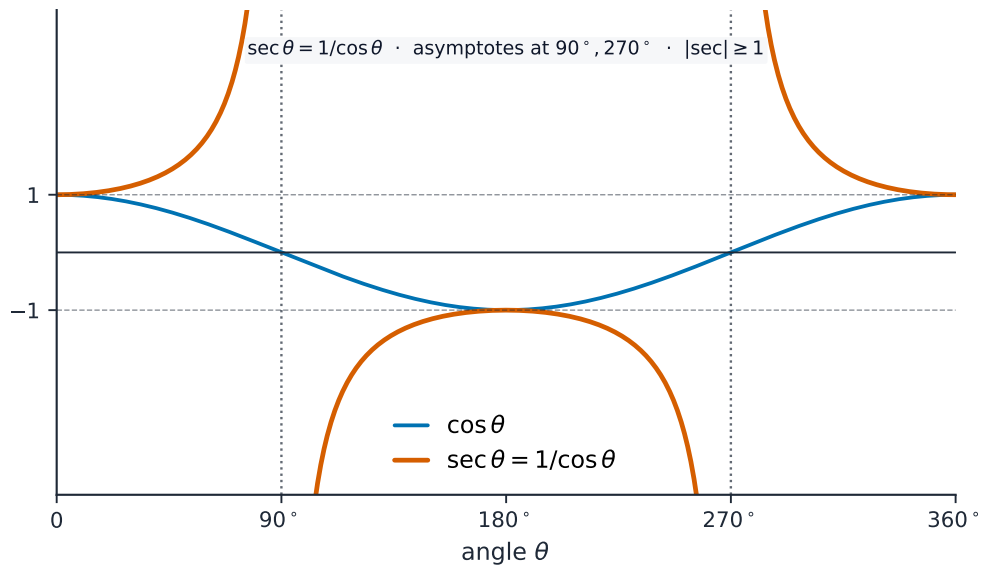
Build the skills to answer exam questions on **trigonometry** in Pure 3 – the reciprocal identities, compound- and double-angle formulae, and the **R-formula**, used in proofs and equations.

**You must be able to:**

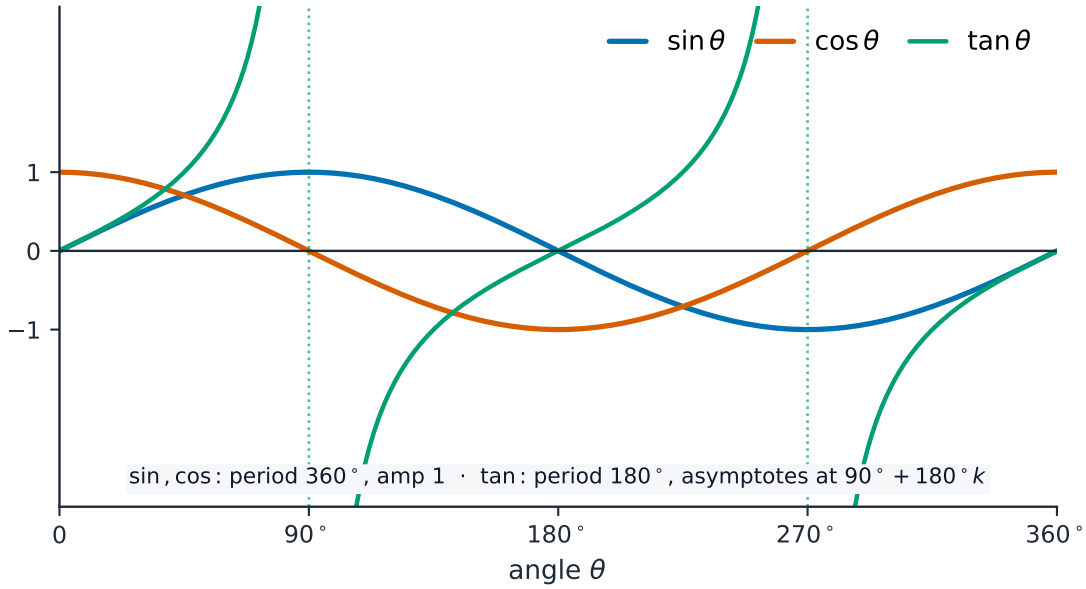
- prove identities using $\sec^2\theta \equiv 1 + \tan^2\theta$ etc.
- expand and use compound/double-angle formulae
- express $a\sin\theta + b\cos\theta$ as $R\sin(\theta \pm \alpha)$ and solve

## 1 Key knowledge and worked examples

### R-formula in an equation




*The reciprocal functions and the identities let you reduce an equation to one function*




*The R-formula rewrites a sum as a single sinusoid*

**Solve $3\sin\theta + 4\cos\theta = 2$ for $0° \leqslant \theta \leqslant 360°$.** Write LHS as $5\sin(\theta + 53.13°)$: $\sin(\theta + 53.13°) = 0.4$, so $\theta + 53.13° = 23.6°$ or $156.4°$ $(+360°)$. Taking those in range gives $\theta = 103.3°$ or $330.5°$.

■ **Key knowledge: Double-angle equations**

Use an identity to put an equation into one trigonometric function, then factor and apply the interval.

■ **Worked example: Double-angle equations**

Solve $\cos 2x = \sin x$ for $0 \leq x < 2\pi$. Since $\cos 2x = 1 - 2\sin^2 x$, the equation becomes $(2\sin x - 1)(\sin x + 1) = 0$. Thus $x = \pi/6, 5\pi/6, 3\pi/2$. Check each branch in the original equation.

## 2 Practice

**2.1** Write $\cos 2\theta$ in terms of $\cos\theta$ only. [1]

**2.2** Find $R$ for $7\sin\theta + 24\cos\theta = R\sin(\theta + \alpha)$. [1]

## 3 Exam-style questions

**3.1** Using $\sin 2\theta = 2\sin\theta\cos\theta$, the value of $\sin 2\theta$ when $\sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{4}{5}$ is: [1]

- ☐ **A** $\frac{12}{25}$
- **B** $\frac{24}{25}$
- **C** $\frac{7}{25}$
- **D** $\frac{6}{5}$

**3.2** (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$, with $R > 0$ and $0° < \alpha < 90°$. [3]

(b) Hence solve $4\sin\theta - 3\cos\theta = 2$ for $0° \leqslant \theta \leqslant 360°$. [4]

**3.3** Prove the identity $\dfrac{\sin 2\theta}{1 + \cos 2\theta} \equiv \tan\theta$. [3]

## Solutions

Each answer shows the steps, then the **result**.

**2.1**

- $\cos 2\theta = 2\cos^2\theta - 1$

**2.2**

- $R = \sqrt{7^2 + 24^2} = 25$

**3.1 B**

- $2 \times \frac{3}{5} \times \frac{4}{5} = \frac{24}{25}$

**3.2**

(a)

- $R = \sqrt{4^2 + 3^2} = 5$; $\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.9°$; $5\sin(\theta - 36.9°)$

(b)

- $\sin(\theta - 36.9°) = 0.4 \Rightarrow \theta - 36.9° = 23.6°$ or $156.4°$; $\theta = 60.5°$ or $193.3°$

**3.3**

- $\sin 2\theta = 2\sin\theta\cos\theta$ and $1 + \cos 2\theta = 2\cos^2\theta$; ratio $= \dfrac{2\sin\theta\cos\theta}{2\cos^2\theta} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$