

2.4 Differentiation (product, quotient, parametric, implicit)

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS parametric • parametric equations • product
• implicit • quotient rule

Objective

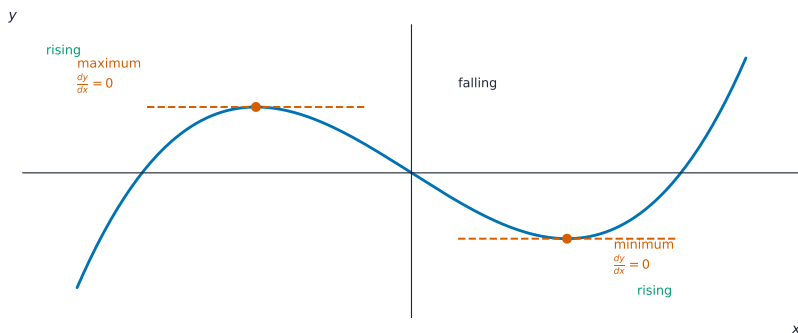
Build the skills to answer exam questions on **differentiation** (e^x , $\ln x$, trig), the **product** and **quotient rules** and **implicit** differentiation. – standard derivatives, and **parametric**

You must be able to:

- differentiate e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$
- apply the product and quotient rules
- differentiate parametric and implicit relations

1 Key knowledge and worked examples

■ Product rule & implicit



stationary points: $\frac{dy}{dx} = 0$ · use $\frac{d^2y}{dx^2}$ or first-derivative sign to classify

A stationary point ($\frac{dy}{dx} = 0$) is still found by differentiating – now with the product/quotient/chain rules

$y = 6x \cos(x^2 + 1)$. Product rule ($u = 6x$, $v = \cos(x^2 + 1)$):

$$\frac{dy}{dx} = 6 \cos(x^2 + 1) - 12x^2 \sin(x^2 + 1).$$

Implicit: for $x^2 + y^2 = 25$, differentiate: $2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$.

■ Key knowledge: A parametric tangent

For parametric equations, $dy/dx = (dy/dt)/(dx/dt)$ when $dx/dt \neq 0$. Both the point and gradient use the same parameter value.

■ Worked example: A parametric tangent

Let $x = t^2 + 1$ and $y = t^3$. At $t = 2$, the point is $(5, 8)$. Since $dx/dt = 2t$ and $dy/dt = 3t^2$, $dy/dx = 3t/2$, giving gradient 3. Therefore the tangent is $y - 8 = 3(x - 5)$.

2 Practice

2.1 Differentiate $y = x^2 e^x$. [2]

2.2 Differentiate $y = \frac{\sin x}{x}$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx}(\ln(3x))$ equals: [1]

- ☐ A $\frac{1}{3x}$
☐ B $\frac{3}{x}$
☐ C $\frac{1}{x}$
☐ D $3 \ln x$

3.2 A curve has parametric equations $x = t^2$, $y = t^3 - 3t$. Find $\frac{dy}{dx}$ in terms of t , and the value of t at the stationary point(s). [4]

3.3 A curve is defined implicitly by $x^2 + xy + y^2 = 7$.

(a) Find $\frac{dy}{dx}$ in terms of x and y . [3]

(b) Find the gradient of the curve at the point $(1, 2)$. [2]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{dy}{dx} = 2xe^x + x^2e^x = xe^x(2 + x)$

2.2

- $\frac{dy}{dx} = \frac{x \cos x - \sin x}{x^2}$

3.1 C

- $\ln(3x) = \ln 3 + \ln x$, so the derivative is $\frac{1}{x}$

3.2

- $\frac{dx}{dt} = 2t$, $\frac{dy}{dt} = 3t^2 - 3$; $\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$; stationary where $3t^2 - 3 = 0 \Rightarrow t = \pm 1$

3.3

(a)

- differentiate: $2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$; $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$

(b)

- at $(1, 2)$: $\frac{dy}{dx} = -\frac{2 + 2}{1 + 4} = -\frac{4}{5}$