1.8 Integration

Name: _____ Class: _____ Date: _____ **Total: 15 marks**

KEY WORDS definite integral • volume of revolution • integration • region

Objective

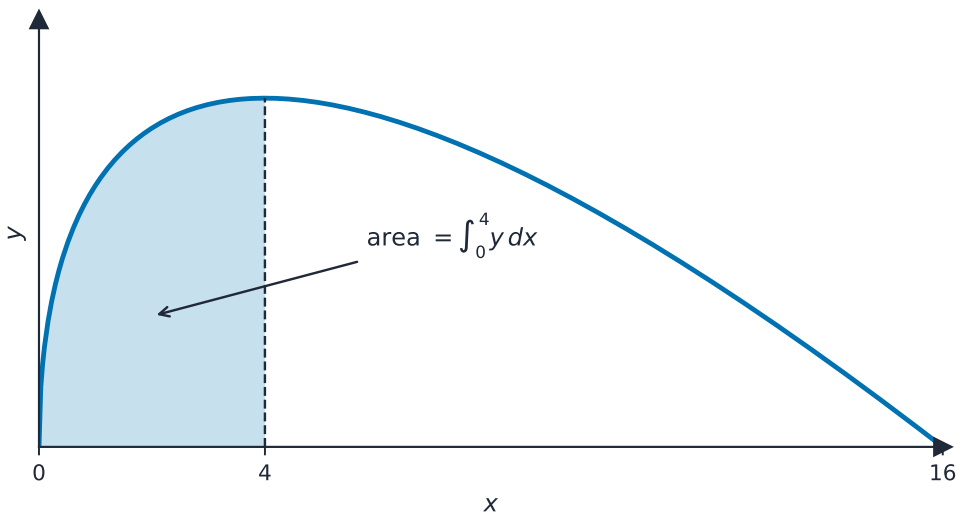
Build the skills to answer exam questions on **integration** – indefinite and **definite integrals**, area under a curve, and **volume of revolution**.

You must be able to:

- integrate powers and $(ax+b)^n$, finding $+C$ from a point
- evaluate a definite integral and find an area
- find a volume of revolution with $V = \pi \int y^2\,dx$

1 Key knowledge and worked examples

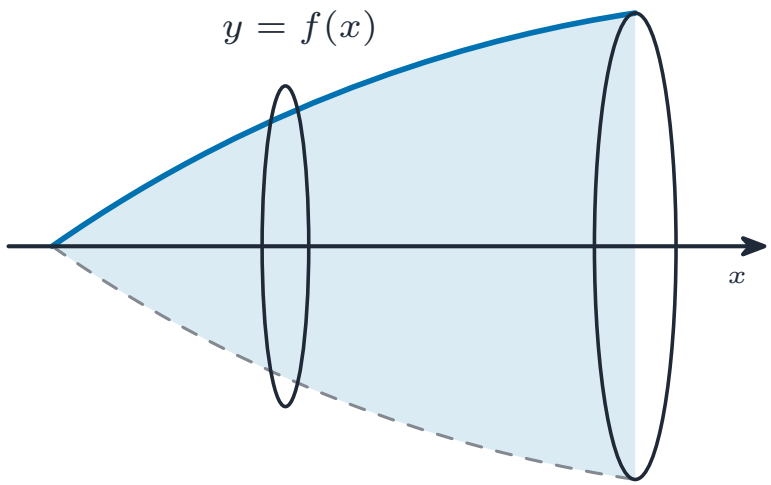
Area and volume



$\int_a^b f(x)\,dx =$ net signed area between $y = f(x)$ and the x-axis

The definite integral $\int_p^q y\,dx$ is the shaded area under the curve

$$\int_0^4 (4x^{1/2} - x)\,dx = \left[\tfrac{8}{3}x^{3/2} - \tfrac{x^2}{2}\right]_0^4 = \tfrac{8}{3}(8) - 8 = \tfrac{40}{3}.$$



rotate the region around the x-axis to sweep a solid

Rotating the region about the x-axis: each slice is a disc of area πy^2, so $V = \pi \int y^2\,dx$

■ **Key knowledge: A volume of revolution**

Rotation about the x-axis produces discs of area πy^2. Integrate the squared function, not the function itself.

■ **Worked example: A volume of revolution**

Rotate $y = x + 1$ from $x = 0$ to $x = 2$ about the x-axis. The volume is $V = \pi \int_0^2 (x+1)^2\,dx$. Thus $V = \pi[x^3/3 + x^2 + x]_0^2 = 26\pi/3$. The units are cubic units; an area integral would answer a different question.

2 Practice

2.1 Find $\displaystyle\int (6x^2 - 4x + 1)\,dx$. [2]

2.2 Evaluate $\int_1^2 \frac{1}{x^2}\,dx$. [2]

3 Exam-style questions

3.1 $\int_0^3 2x\,dx$ equals: [1]

- ☐ **A** 6
- **B** 9
- **C** 12
- **D** 18

3.2 A curve has $\frac{dy}{dx} = 3x^2 - 4x$ and passes through $(2, 5)$. Find the equation of the curve. [4]

3.3 The region R is bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$.

(a) Find the area of R. [3]

(b) Find the volume when R is rotated through 360° about the x-axis. [3]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $2x^3 - 2x^2 + x + C$

2.2

- $\left[-\dfrac{1}{x}\right]_1^2 = -\dfrac{1}{2} - (-1) = \dfrac{1}{2}$

3.1 B

- $[x^2]_0^3 = 9$

3.2

- $y = \displaystyle\int (3x^2 - 4x)\,dx = x^3 - 2x^2 + C$; use $(2, 5)$: $5 = 8 - 8 + C \Rightarrow C = 5$; $y = x^3 - 2x^2 + 5$

3.3

(a)

- $\displaystyle\int_0^4 x^{1/2}\,dx = \left[\tfrac{2}{3}x^{3/2}\right]_0^4 = \tfrac{2}{3}(8) = \dfrac{16}{3}$

(b)

- $V = \pi \displaystyle\int_0^4 (\sqrt{x})^2\,dx = \pi \int_0^4 x\,dx = \pi \left[\tfrac{x^2}{2}\right]_0^4 = 8\pi$