1.6 Series

Name: _____ Class: _____ Date: _____ **Total: 13 marks**

KEY WORDS progression • arithmetic • geometric progression • sum to infinity • common ratio

Objective

Build the skills to answer exam questions on **series** – the **binomial expansion**, **arithmetic** and **geometric progressions**, and the **sum to infinity**.

You must be able to:

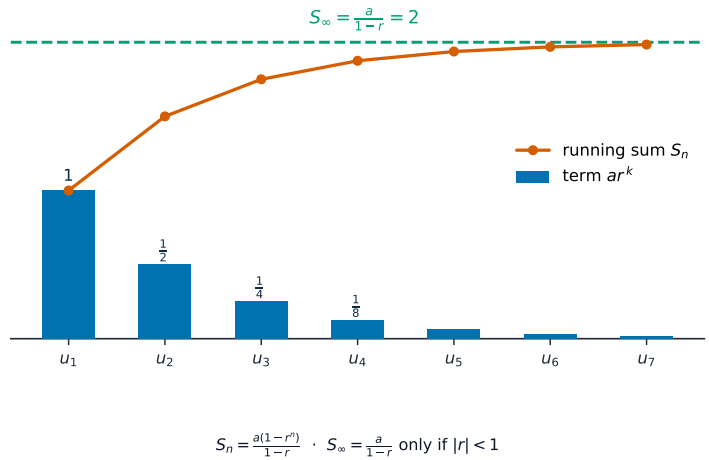
- expand $(a+b)^n$ and find a specific term/coefficient
- use the AP and GP formulas for the nth term and the sum
- use $S_\infty = \dfrac{a}{1-r}$ when $|r| < 1$

1 Key knowledge and worked examples

■ **Binomial & GP**

First three terms of $(2-px)^5$:

$$2^5 + \binom{5}{1}2^4(-px) + \binom{5}{2}2^3(-px)^2 = 32 - 80px + 80p^2x^2.$$



A convergent GP ($|r| < 1$): the terms shrink and the running sum approaches

$$S_\infty = \frac{a}{1-r}$$

A GP has third term 18 and $S_3 = 26$, with $r < 0$. From $ar^2 = 18$ and $a(1+r+r^2) = 26$: $8r^2 - 18r - 18 = 0$, so $r = -\frac{3}{4}$, $a = 32$, and $S_\infty = \dfrac{32}{1+\frac{3}{4}} = \dfrac{128}{7}$.

■ **Key knowledge: An arithmetic sum**

Distinguish the last term from the number of terms. For an arithmetic progression, $u_n = a + (n-1)d$ and $S_n = n(a+u_n)/2$.

■ **Worked example: An arithmetic sum**

The progression $7, 11, 15, \ldots$ ends at 83. From $83 = 7 + 4(n-1)$, $n = 20$. Therefore $S_{20} = 20(7+83)/2 = 900$. Checking the last term first prevents using 83 as the term count.

2 Practice

2.1 Find the 10th term of the AP $3, 7, 11, \ldots$. [2]

2.2 State the condition on r for a GP to have a sum to infinity. [1]

3 Exam-style questions

3.1 The coefficient of x^2 in $(1+2x)^6$ is: [1]

- ☐ **A** 12
- **B** 30
- **C** 60
- **D** 15

3.2 The first term of a GP is 24 and the sum to infinity is 40.

(a) Find the common ratio. [3]

(b) Find the sum of the first three terms. [2]

3.3 An AP has first term 5 and common difference 3. The sum of the first n terms is 440. Find n. [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $u_{10} = a + 9d = 3 + 9(4) = 39$

2.2

- $|r| < 1$

3.1 C

- $\binom{6}{2}(2)^2 = 15 \times 4 = 60$

3.2

(a)

- $S_\infty = \dfrac{a}{1-r} = 40 \Rightarrow \dfrac{24}{1-r} = 40 \Rightarrow 1 - r = 0.6;\ r = 0.4$

(b)

- $S_3 = \dfrac{24(1-0.4^3)}{1-0.4} = \dfrac{24(0.936)}{0.6} = 37.44$

3.3

- $S_n = \dfrac{n}{2}(2a + (n-1)d) = 440 \Rightarrow \dfrac{n}{2}(10 + 3(n-1)) = 440;\ 3n^2 + 7n - 880 = 0;$ $(3n + 55)(n - 16) = 0;\ n = 16$ (reject negative)