

1.4 Circular measure

Name: _____ Class: _____ Date: _____

Total: 13 marks

KEY WORDS radian • sector • arc length • sector area • segment

Objective

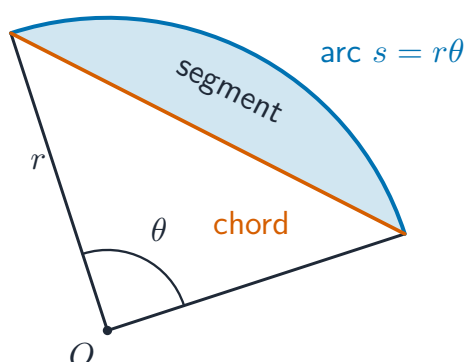
Build the skills to answer exam questions on **circular measure** – **radians**, **arc length**, **sector area** and the area of a **segment**.

You must be able to:

- convert between degrees and radians
- use $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ (with θ in radians)
- find a segment area as sector minus triangle

1 Key knowledge and worked examples

■ Sector and segment



The shaded segment is the part of the sector between the chord and the arc

For a sector (angle θ in radians): $s = r\theta$, $A = \frac{1}{2}r^2\theta$. Segment = $\frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \frac{1}{2}r^2(\theta - \sin \theta)$.

A sector has $r = 6$, $\theta = \frac{2}{3}\pi$. Arc = $6 \cdot \frac{2}{3}\pi = 4\pi$; area = $\frac{1}{2}(36)\frac{2}{3}\pi = 12\pi$.

■ **Key knowledge: Segment area**

A minor segment is the sector area minus the triangle area. Use radian mode for both the angle and the sine.

■ **Worked example: Segment area**

For $r = 4$ cm and $\theta = \pi/3$, the segment area is $A = \frac{1}{2}r^2(\theta - \sin \theta)$. Substitution gives $A = 8(\pi/3 - \sqrt{3}/2) = 8\pi/3 - 4\sqrt{3} \approx 1.45 \text{ cm}^2$. The result is smaller than the sector area, as required.

2 Practice

2.1 Convert 120° to radians (exact). [1]

2.2 A sector has $r = 5$ cm and $\theta = 0.8$ rad. Find its arc length. [1]

3 Exam-style questions

3.1 The area of a sector with $r = 4$ and $\theta = \frac{\pi}{6}$ is: [1]

- ☐ **A** $\frac{2\pi}{3}$
B $\frac{4\pi}{3}$
C $\frac{\pi}{3}$
D 2π

3.2 A sector OAB has centre O , radius 10 cm and angle $AOB = 1.2$ radians.

(a) Find the perimeter of the sector. [3]

(b) Find the area of the **segment** between the chord AB and the arc. [3]

3.3 A sector has perimeter 20 cm and area 16 cm^2 . Show that the radius r satisfies $r^2 - 10r + 16 = 0$, and hence find the two possible radii. [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $120 \times \frac{\pi}{180} = \frac{2\pi}{3}$

2.2

- $s = r\theta = 5 \times 0.8 = 4 \text{ cm}$

3.1 B

- $A = \frac{1}{2}(16)\frac{\pi}{6} = \frac{4\pi}{3}$

3.2

(a)

- $\text{arc} = 10 \times 1.2 = 12$; perimeter $= 12 + 10 + 10 = 32 \text{ cm}$

(b)

- $\text{segment} = \frac{1}{2}(10^2)(1.2 - \sin 1.2) = 50(1.2 - 0.9320) = 50(0.2680) = 13.4 \text{ cm}^2$

3.3

- perimeter: $2r + r\theta = 20$; area: $\frac{1}{2}r^2\theta = 16 \Rightarrow r\theta = \frac{32}{r}$. Substitute: $2r + \frac{32}{r} = 20 \Rightarrow 2r^2 - 20r + 32 = 0 \Rightarrow r^2 - 10r + 16 = 0$. Factorise $(r - 2)(r - 8) = 0 \Rightarrow r = 2$ or $r = 8$