

1.1 Quadratics

Name: _____ Class: _____ Date: _____

Total: 16 marks**KEY WORDS** quadratic

- simultaneous equations

• quadratic equation

- real roots

• discriminant

Objective

Build the skills to answer exam questions on **quadratics** – completing the square, the **discriminant**, solving equations and inequalities, and simultaneous equations.

You must be able to:

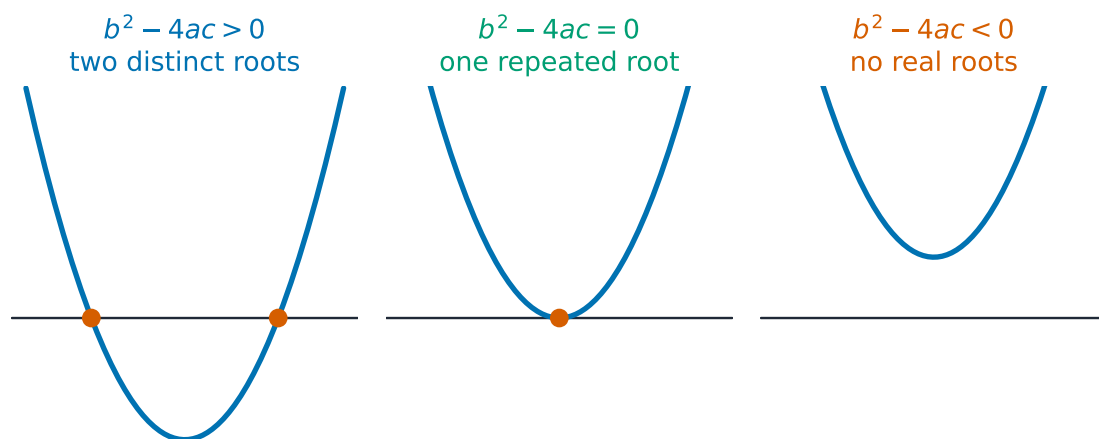
- write $ax^2 + bx + c$ in completed-square form $a(x + p)^2 + q$
- use $b^2 - 4ac$ to decide the number of real roots
- solve quadratic equations and inequalities
- solve a linear-quadratic pair by substitution

1 Key knowledge and worked examples

■ Completing the square & the discriminant

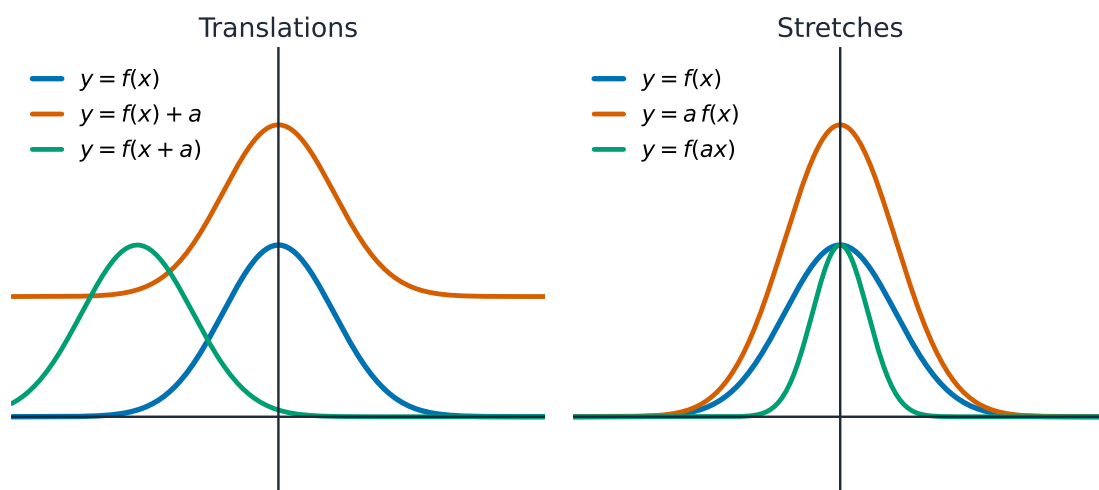
Write $2x^2 - 12x + 5$ as $a(x + p)^2 + q$.

$$2x^2 - 12x + 5 = 2(x^2 - 6x) + 5 = 2((x - 3)^2 - 9) + 5 = 2(x - 3)^2 - 13.$$



$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (\text{real roots need } b^2 - 4ac \geq 0)$$

The sign of $b^2 - 4ac$ decides how many times the parabola meets the x -axis



$f(x) + a$ moves up · $f(x + a)$ moves left · $af(x)$ stretch y · $f(ax)$ compress x if $a > 1$

Completing the square shifts and stretches the parabola

Find the values of k for which $x^2 + kx + 9 = 0$ has two distinct real roots.

Two distinct roots need $b^2 - 4ac > 0$: $k^2 - 36 > 0 \Rightarrow (k - 6)(k + 6) > 0 \Rightarrow k < -6$ or $k > 6$.

■ Key knowledge: A quadratic inequality

For an upward-opening quadratic with two roots, the expression is negative between the roots. Include the roots only for a non-strict inequality.

■ **Worked example: A quadratic inequality**

Solve $x^2 - 5x + 6 \leq 0$. Factor: $f(x) = (x - 2)(x - 3)$. The roots are 2, 3 and the leading coefficient is positive. Therefore $f(x) \leq 0$ gives $2 \leq x \leq 3$. Testing $x = 0$ confirms that the outside region is positive.

2 Practice

2.1 Express $x^2 + 8x + 3$ in the form $(x + p)^2 + q$. [2]

2.2 State the condition on the discriminant for a quadratic to have **no** real roots. [1]

3 Exam-style questions

3.1 The equation $2x^2 - 4x + c = 0$ has a repeated root. The value of c is: [1]

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A 0

B 1

C 2

D 4

3.2 Express $3x^2 + 12x + 7$ in the form $a(x + b)^2 + c$ and hence write down the least value of the expression. [4]

3.3 Find the set of values of k for which the line $y = kx - 2$ meets the curve $y = x^2 - 3x + 2$ at two distinct points. [4]

3.4 Solve the simultaneous equations $y = 2x - 1$ and $x^2 + y^2 = 2$. [4]

Solutions

Each answer shows the steps, then the **result**.

2.1

- $(x + 4)^2 - 13$

2.2

- $b^2 - 4ac < 0$

3.1 C

- Repeated root needs $b^2 - 4ac = 0$: $16 - 8c = 0 \Rightarrow c = 2$

3.2

- $3(x^2 + 4x) + 7 = 3((x + 2)^2 - 4) + 7 = 3(x + 2)^2 - 5$; least value = -5 (when $x = -2$)

3.3

- set equal: $x^2 - 3x + 2 = kx - 2 \Rightarrow x^2 - (3 + k)x + 4 = 0$; two points need $b^2 - 4ac > 0$:
 $(3 + k)^2 - 16 > 0$; $(k + 7)(k - 1) > 0$; $k < -7$ or $k > 1$

3.4

- substitute: $x^2 + (2x - 1)^2 = 2 \Rightarrow 5x^2 - 4x - 1 = 0$; $(5x + 1)(x - 1) = 0 \Rightarrow x = -\frac{1}{5}$ or $x = 1$; $y = 2x - 1$ gives $(1, 1)$ and $(-\frac{1}{5}, -\frac{7}{5})$