

Exercise walk-through

6.5

Hypothesis tests

A-Level Mathematics

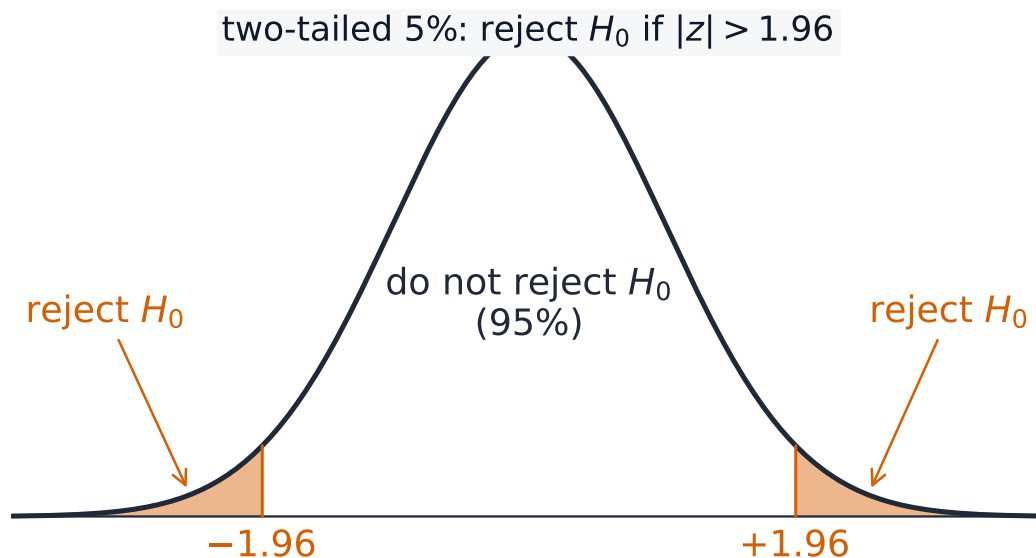
Hypothesis tests

Null H_0 vs **alternative** H_1 ; pick a **significance level** .

Test statistic in the **rejection region** $\Rightarrow$ reject H_0 .

Type I : reject a true H_0 ; **Type II** : accept a false H_0 .

Hypothesis tests, in detail



A two-tailed test at 5% rejects H_0 only if the test statistic falls in a shaded tail beyond ± 1.96 .

The parts of a hypothesis test

a hypothesis test

uses sample data to judge a claim

the null hypothesis

H_0 – the claim being tested, always an equality

the alternative

H_1 – what you conclude if H_0 is rejected; one-tailed or two-tailed

the significance level

often 5% – fixed **before** you look at the data

the test statistic

worked out from the data, and compared with the critical region

the conclusion

reject or do not reject H_0 – **in context**, and never “accept H_0 ”

Not rejecting is not proving. The test can only say the evidence was insufficient, which is why the wording of the conclusion is marked.

The parts of a test, and the two ways it fails

The hypotheses

H_0 states the value being tested; the **alternative hypothesis** H_1 states what you would accept instead, and its form decides one tail or two.

What is estimated

A sample gives **unbiased estimates** of the mean and variance – and, for a proportion, of the **population proportion**.

Type I error

A **type I error** : rejecting H_0 when it was true. Its probability is the significance level.

Type II error

A **type II error** : failing to reject H_0 when it was false. Lowering the significance level raises this one.

You never prove H_0 . The conclusion is either “sufficient evidence to reject” or “insufficient evidence” – in the context of the question.

Worked example – a Poisson probability

$X \sim \text{Po}(3)$. Find $P(X = 2)$.

$$P(X = 2) = e^{-3} \frac{3^2}{2!} = e^{-3} \times 4.5 = 0.224$$

For a Poisson variable the mean **and** variance both equal λ . Use $P(X = r) = e^{-\lambda} \frac{\lambda^r}{r!}$.

Exercises 2.1-2.2

2.1 State the meaning of a **Type I error**.

[1]

2.2 Give the critical z value for a one-tailed test at the 5% level.

[1]

Exercises 2.1·2.2

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[1]

2.1

- rejecting H_0 when it is in fact true

2.2 Give the critical z value for a one-tailed test at the 5% level.

[1]

2.2

- 1.645

Exercise 3.1

3.1 In a test at the 5% level, H_0 is rejected when the p -value is:

[1]

☐

A greater than 0.05

B less than 0.05

C equal to 1

D negative

Exercise 3.1

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[1]

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3.1 B

- Reject H_0 when the p -value is below the significance level 0.05

Exercise 3.2 (a)

3.2 A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(a) State H_0 and H_1 , and whether the test is one- or two-tailed. [2]

Exercise 3.2 (a)

3.2 A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(a) State H_0 and H_1 , and whether the test is one- or two-tailed. [2]

(a)

- $H_0: \mu = 500$, $H_1: \mu > 500$; one-tailed

Exercise 3.2 (b)

3.2 A machine should fill bottles to a mean of 500 ml with $\sigma = 6$ ml. A sample of 36 bottles has mean $\bar{x} = 502$ ml. Test at the 5% level whether the mean has increased.

(b) Carry out the test and state your conclusion.

[4]

Exercise 3.2 (b)

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(b) Carry out the test and state your conclusion.

[4]

(b)

- $z = \frac{502 - 500}{6/\sqrt{36}} = \frac{2}{1} = 2$; critical value 1.645; since $2 > 1.645$, reject H_0 : there is evidence the mean has increased

Exercise 3.3 (a)

3.3 A coin is tossed 20 times to test $H_0 : p = 0.5$ against $H_1 : p > 0.5$ at the 5% level, using the number of heads $X \sim B(20, 0.5)$. The critical region is $X \geq 15$.

(a) Find $P(X \geq 15)$ under H_0 and explain why $X \geq 15$ is used. [3]

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(a)

- $P(X \geq 15) = 0.0207$ (from tables); this is the smallest "tail" region with probability ≤ 0.05 , so it is the 5% critical region

Exercise 3.3 (b)

3.3 A coin is tossed 20 times to test $H_0 : p = 0.5$ against $H_1 : p > 0.5$ at the 5% level, using the number of heads $X \sim B(20, 0.5)$. The critical region is $X \geq 15$.

(b) State what a Type I error would mean in this context.

[1]

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(b) State what a Type I error would mean in this context. [1]

(b)

- concluding the coin is biased towards heads ($p > 0.5$) when in fact it is fair